

## Effect of Nuclear vibrations on Shell Model parameters

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### Introduction

Single-particle shell model is a microscopic approach to study nuclear structure. One expects Single-particle shell model to represent the best explanation of spherical nuclei. In the present work, single-particle energies obtained for double-magic nuclei  $^{208}_{82}\text{Pb}$ ,  $^{132}_{50}\text{Sn}$ ,  $^{56}_{28}\text{Ni}$ ,  $^{48}_{20}\text{Ca}$ ,  $^{40}_{20}\text{Ca}$ ,  $^{16}_8\text{O}$ ,  $^8_2\text{He}$ ,  $^4_2\text{He}$  and magic-submagic nuclei  $^{90}_{40}\text{Zr}$ ,  $^{88}_{38}\text{Sr}$ ,  $^{68}_{28}\text{Ni}$ ,  $^{66}_{28}\text{Ni}$ ,  $^{36}_{16}\text{S}$ ,  $^{14}_6\text{C}$  and  $^{12}_6\text{C}$ , which compared with experimental energies evaluated from stripping and pick-up reactions. The corresponding Schrödinger equation for phenomenological Woods-Saxon potential calculated numerically as an eigenvalue differential equation due to fourth order Rung-Kutta method. Then potential depth, surface diffuseness and spin-orbit strength parameters were fitted for six first nuclei to obtain a set of optimized parameters for proton and neutron part of each potential. Final results were not desirable completely. At the end, we studied the probability of getting better results among including the particle-core coupling model. Though single-particle shell model explains spherical nuclei the best, but it is far reaching from ideal model which predicts all experimental details.

### Shell Model Calculations

We have studied different methods to solve the radial Schrödinger equation [1] with Woods-Saxon Potential,

$$\left[ -\frac{\hbar^2}{2m_N} \left( \frac{1}{r} \frac{d^2}{dr^2} r - \frac{l(l+1)}{r^2} \right) + V_{ws} + V_c + V_{LS} L.S \right] R(r) = E R(r) \quad (1)$$

$$V_{ws} = -\frac{V_0}{1 + \exp[(r-R)/a]} \quad (2)$$

Which spin-orbit potential [2] is indicated as eq. (3) and coulomb potential (4) acts on proton wave functions only.

$$V_{LS}(r) = V_{LS}^{(0)} \left( \frac{r_0}{\hbar} \right)^2 \frac{1}{r} \left[ \frac{d}{dr} \frac{1}{1 + e^{(r-R)/a}} \right] \quad (3)$$

$$V_c(r) = \frac{Ze^2}{4\pi\epsilon_0} \begin{cases} \frac{1}{R} \left[ \frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} \right] & r \leq R \\ \frac{1}{r} & r > R \end{cases} \quad (4)$$

Since the analytical solution of eq. (1) is not available, the eigen energies can be obtained due to numerical solutions such as rung-kutta method.

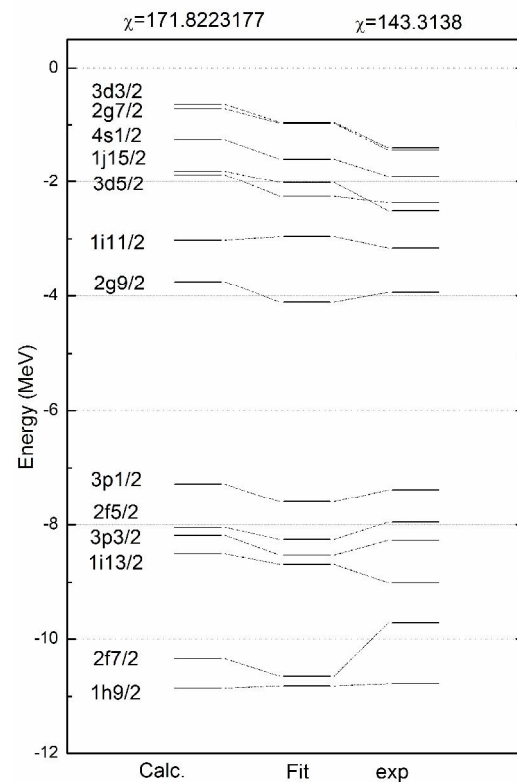


Figure 1. Calculated, fitted and experimental neutron energies for  $^{208}\text{Pb}$ .

Then the calculated eigenvalues have been fitted to experimental energies to obtain the optimized Woods-Saxon parameters. Fig.1 exhibited a typical fit for neutrons of  $^{208}\text{Pb}$  nucleus. the  $\chi$  quantity which indicated in this figure is calculated due to eq. (5).

$$\chi = \left[ \sum_{i=1}^n \frac{(E_{calc}^i - E_{ex}^i)^2}{\sigma^2} \right]^{1/2} \quad (5)$$

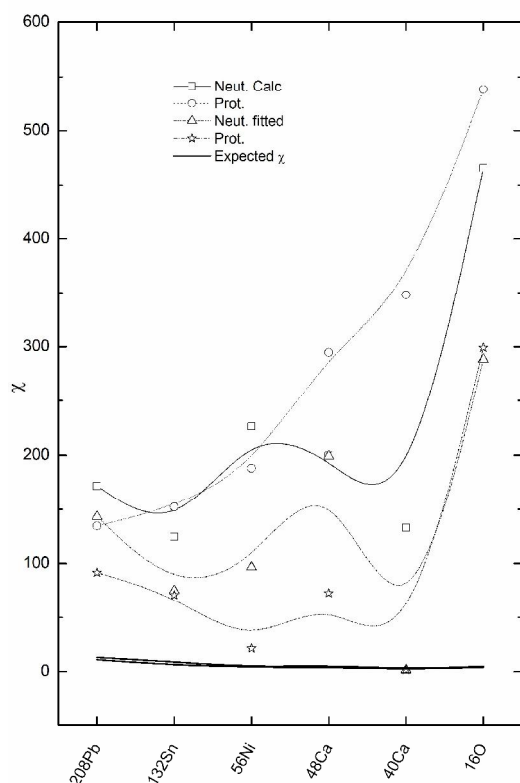


Figure 2  $\chi$  quantities for 6 nuclei

It is apparent that the  $\chi$  quantities are not even near those which expected for an ideal fitting process. To make the consistency between theory and experiment better, we can include corrections such as particle-core coupling.

### Particle-core coupling

The essential assumption in the former section is considering the states in the excitation spectra of a nucleus to be pure single-particle or

single-hole states. This assumption has been questioned and a coupling term [3] introduced to include the vibration modes of double-magic core in the excited states.

$$H_{coupl} = (2\lambda + 1)^{1/2} k_{\lambda}(r)(Y_{\lambda}\alpha_{\lambda})_0 \quad (6)$$

In fact, single-particle states and vibration modes both contribute in the excited states of a closed-shell  $\pm$  one nucleon nucleus.

The wave function can be written as  $|(\lambda j')j\rangle$  which  $\lambda$ ,  $j'$  and  $j$  represent angular momentum of vibrational quanta (see table 1), single-particle state spin and total spin of excited state correspondingly.

Table 1. Collective states of  $^{208}\text{Pb}$  [4].

E (MeV)	$\lambda$	$\langle\alpha_{\lambda}\rangle$
2.61	$3^-$	0.046
3.21	$5^-$	0.017
4.08	$2^+$	0.025
4.32	$4^+$	0.024
4.42	$6^+$	0.015

### Results

The matrix elements (6) for  $h9/2$  and  $f7/2$ , two proton states of  $^{208}\text{Pb}$ , were calculated and the energy displacements were obtained about 0.1 and 0.4 MeV for corresponding states.

Then single-particle energies can be considered as parameters, energies of vibrational modes  $\hbar\omega_{\lambda}$  and vibrational amplitudes  $\langle\alpha_{\lambda}\rangle$  can be taken from experimental data (see table 1). Therefore a set of single-particle energies can be obtained from the fitting process which can be applied to optimize Woods-Saxon parameters again.

### References

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