Comparative study of two parameter formulae for A=120-200 mass region

H.M. Mittal[1],* Vidya Devi[1], and J. B. Gupta[2]
[1]Dr B. R. Ambedkar, National Institute of Technology Jalandhar-144011, INDIA and
[2]Ramajs College, University of Delhi-110007, INDIA

Introduction

Our understanding of nuclear structure is framed within the context of a number of idealized benchmark. These include the axial rotor [1], the anharmonic vibrator [2] and γ-soft deformed nuclei [3, 4]. The simplest expression for rotational spectra is

$$E = \frac{\hbar^2}{2\Im} J(J+1). \tag{1}$$

where $\Im$ is moment of inertia and J be the spin of nuclei. Holmberg and Lipas [5] suggested the simple two parameter formula. They noted that the Moment of Inertia (MI) of the deformed nuclei increases with level energy linearly, i.e.

$$\Im(J) = a + bE \tag{2}$$

By substituting eq.(2) in eq.(1)

$$E = a\left[\sqrt{1+bJ(J+1)} - 1\right] \tag{3}$$

should be obtained. Holmberg observe that ab formula gives the better agreement for well deformed nuclei but, for softer nuclei they had to calculate a, b from 6^+, 8^+ energies to get reasonable agreement. Later Zeng et al. [6] illustrated the non linearity of the relation between $\Im$ and E in the rotational spectra for low and high spins. However they derived a new relation between $\Im$ and E

$$\Im = \frac{1}{2ab}\left(\sqrt{1+\frac{2}{a}E} + 1\right). \tag{4}$$

by rewriting eq.(3) and comparing it with eq.(1), and by substituting eq.(4) into eq.(1) they obtained pq formula

$$\begin{aligned} E(J) = a(&\{(bJ(J+1)/2)^2 + [(bJ(J+1)/2)^4 \\ &+ (bJ(J+1)/3)^3]^{\frac{1}{2}}\}^{\frac{1}{3}} + \{(bJ(J+1)/2)^2 \\ &- [(bJ(J+1)/2)^4 + (bJ(J+1)/3)^3]^{\frac{1}{2}}\}^{\frac{1}{3}}) \end{aligned} \tag{5}$$

The pq formula gives goods results for well deformed nuclei when fitted for 2^+, 4^+ and 6^+, 8^+ energy level but for soft nuclei the fit is worse. The fits given by the pq formula are far better than those given by ab formula. Further Brentano [7] noted that MI depends upon the spin (J) and the energy (E)

$$\Im = \Im_0(1 + aJ + bE). \tag{6}$$

By dropping the energy-dependent term in this expression and then putting this eq.(6) in eq.(1) Brentano got two parameter formula

$$E = \frac{1}{\Im_0(1+aJ)} J(J+1). \tag{7}$$

Comparison of Brentano formula which is also called soft rotor formula (SRF) with other two parameter formulae shows that SRF gives good agreement for well deformed nuclei, but for soft nuclei the agreement is not good for both 2^+, 4^+ and 6^+, 8^+ levels. Gupta et al. [8] studied the single term expression of ground band level energies of a soft rotor. Here the concept of an arithmetic mean of the two terms used in Bohr-Mottelson expression was replaced by the concept of a geometric mean in the form of the power law $E_J = aJ^b$. The purpose of this note is to study the two parameter formula for A=120-200 mass region nuclei, to show this two parameter

*Electronic address: mittalhm@nitj.ac.in

formula is also good to study the ground band energy for A=120-200 mass region nuclei as the other two parameter formulae such as ab, pq and Brentano. For this study the experimental data is taken from [9].

Results and Discussion

For the stringently test of two parameter formula $E_J = aJ^b$ for well deformed nuclei and soft nuclei, we carried out the least square fit of power law to calculate the value of constants a and b . In Fig.1 we compare the various two parameter formulae for ^{122}Xe nuclei. When we fit J=2^+, 4^+ energy level then ab, pq and SRF shows irregularity in there theoretical energy values but, power law shows close agreement with experimental values. Similarly when we fit J=6^+, 8^+ energy level then SRF and power law both shows good agreement but ab and pq shows irregularity. In power law the root mean square deviation (RMSD) lies within 5% for the worst cases.

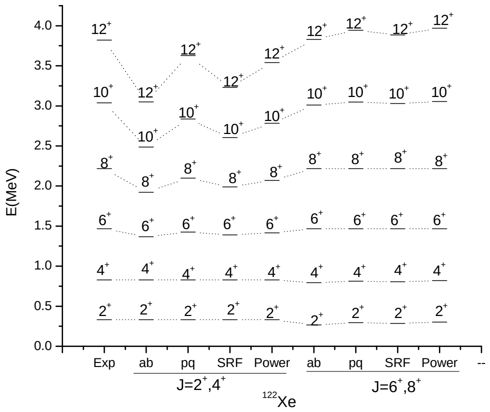

FIG. 1: Comparison between experimental energy with ab, pq, SRF and two parameter formulae.

Power law gives good agreement for deformed and soft nuclei by fitting either 2^+, 4^+ or 6^+, 8^+ energy levels.

Conclusion

To summarize, we describe the two parameter formula, which is applicable for both deformed and soft nuclei. This two parameter formula shows good agreement in energy values by fitting 2^+, 4^+ either 6^+, 8^+ energy levels. The pq, ab and SRF formula gives goods results for well deformed nuclei when fitted for 2^+, 4^+ and 6^+, 8^+ energy level but for soft nuclei the fit is worse. An alternate expression for the energies in the rotation-vibration $K^\pi = 0^+$ ground bands in deformed nuclei is suggested and applied for the study of A=120-200 mass region. This two parameter formula is also successful to explain ground band energies as compared to other two parameter models.

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