Wigner-Kirkwood Approach for Mass Calculation: Effect of higher order correction terms

A. Bhagwat[1],* J. P. Maharana[2], and Y. K. Gambhir[3]
[1]*Dept. of Physics, IIT Gandhinagar, Ahmedabad 382424, India*
[2]*Computer Centre, IIT Bombay, Powai, Mumbai 400076, India and*
[3]*Dept. of Physics, IIT Bombay, Powai, Mumbai 400076, India.*

Introduction

Calculation of nuclear masses is still being pursued vigorously by a number of groups around the globe. There are primarily two distinct approaches to calculate masses: a) the microscopic nuclear models based on the density functional theory (like, Skyrme Hartree Fock Bogoliubov [1] or Relativistic Mean Field (RMF) models [2]) b) microscopic - macroscopic (Mic - Mac) models [3]. The Mic - Mac models typically yield better than ≈0.7 MeV rms deviation in the masses.

A number of Mic - Mac calculations with varying degree of success are available in the literature (see, for example, [3–5]). The basic idea behind such models is to calculate the bulk part of binding energy using the liquid drop model and the shell effects using the well known Strutinsky smoothing scheme [6, 7]. All these models agree reasonably well with each other and with the experiment, but deviate widely in the regions far away from the valley of stability.

The Mic - Mac models are based on the well - known Strutinsky theorem. According to this, the nuclear binding energy, hence the mass can be written as sum of a smooth part, and an oscillatory part. The latter is often referred to as shell correction. The smooth part is normally taken from the liquid drop models of different degrees of sophistication. The biggest uncertainties arise in the calculation of shell corrections. Usually, it is achieved by the well established Strutinsky scheme. This technique of calculating the smoothed energies runs into practical difficulties for finite potentials. Alternatively, one may carry out direct averaging of the Hamiltonian, within the Wigner - Kirkwood (WK) averaging scheme [8–10]. This approach so far has not been employed in the calculation of nuclear masses, primarily due to the mathematical complexity involved in that. This has recently been achieved [11] by carrying out the WK expansion explicitly upto the fourth order in $\hbar$. Calculations for all the spherical nuclei appearing in the periodic table have also been reported [11] and it has been found that the Mic Mac approach with WK averaging method yields rms deviation of 630 keV [11], which is indeed, promising.

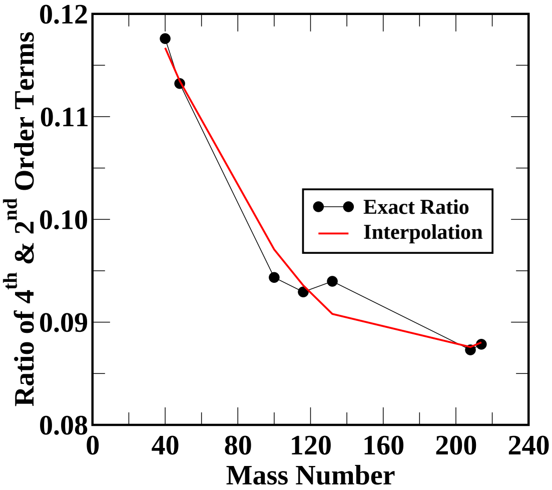

FIG. 1: The ratio of fourth and second order correction terms, along with the quadratic interpolation.

Details of the Calculations

It turns out that the fourth order WK calculations are very involved, and time consum-

*Electronic address: ameeya@kth.se

ing. It is therefore interesting and important to check if the effects of fourth order contributions can be incorporated in an approximate way [12]. In this short communication, we examine this problem. To achieve that, we analyse the ratio of fourth and second order correction terms in Fig. (1), for ^{40,48}Ca, ^{100,116,132}Sn and ^{208,214}Pb. It is seen that the ratio approximately follows a quadratic dependence on mass number, as revealed by regression curve plotted in the same figure. A more detailed analysis reveals that the shell corrections calculated by taking fourth order effects approximately into account, agrees closely with the corresponding exact values, and the resulting ground state masses are reproduced with rms deviation of around 640 keV [11]. More detailed work in this direction is in progress, particularly for inclusion of deformation degrees of freedom in this analysis.

It appears that it may be possible to include the fourth order contributions approximately in the results of the appropriately modified second order calculations.

Acknowledgments

The authors thank Ramon Wyss, P. Schuck, Xavier Vinyes and Mario Centelles for collaborating in part of this work.

References

[1] S. Goriely *et al.*, Phys. Rev. C **66**, 024326 (2002) and references cited therein.

[2] G. A. Lalazissis *et al.*, At. Data Nucl. Data Tables **71**, 1 (1999).

[3] P. Möller *et al.*, At. Data Nucl. Data Tables **59** 185 (1995).

[4] K. Pomorski and J. Dudek, Phys. Rev. C **67**, 044316 (2003) and references cited therein.

[5] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **A601** 141, (1996).

[6] V. M. Strutinsky, Nucl. Phys. **A95**, 420 (1967).

[7] G. G. Bunatian *et al.*, Nucl. Phys. **A188**, 225 (1972).

[8] B. K. Jennings *et al.*, Nucl. Phys. **A253**, 29 (1975).

[9] M. Brack and R. K. Bhaduri, Semiclassical Physics, Addison - Wesley Publishing Co., 1997.

[10] M. Centelles *et al.*, Phys. Rev. C **74**, 034332 (2006) and references cited therein.

[11] A. Bhagwat, R. Wyss, M. Centelles, X. Vinyes, P. Schuck, communicated for publication.

[12] A. Bhagwat and P. Schuck, to be communicated for publication.