S-matrix based unified calculation of Q - values and half lives of α – decay of super heavy elements

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Introduction

The production of a large number of super heavy nuclei undergoing α- decay have generated much interest in the calculation of their decay properties [1-4]. In most studies, the half lives($\tau_{1/2}$) are calculated using a suitable potential between α and the daughter nucleus and then computing the decay probabilities employing the experimental *Q* values with WKB type approximations. In Refs. [2,3] we have compared the S-matrix (SM) method with the WKB method for a set of α-decaying nuclei and discussed the limitations of the latter. However, the potential used there has Woods Saxon form factor obtained by approximating the double folded potentials generated by relativistic mean field (RMF) based density distributions. Since then, larger set of experimental data for a number of α-decaying nuclei have been studied based on empirical formulae [5,6]. It is therefore, worthwhile to carry out a more extended and exhaustive study of these systems in the SM based unified approach and using the exact numerically available RMF based potentials for α-daughter nucleus systems and their comparison with the corresponding experimental and empirical formulae results. It is known that even a slight change in potential can substantially change the half life and hence the desirability of using exact numerical RMF generated potential rather than their approximation using Woods – Saxon form. This may provide a tacit support and perhaps a theoretical justification for the empirical formulae. In addition, it imposes a stringent test for the RMF based α-nucleus potentials.

Theory and Calculations

We briefly summarize the method of unified calculation of *Q* values and widths based on the SM approach. In this approach α-decay is assumed to be the decay of a positive energy resonance or quasi-bound state of the α-daughter nucleus system governed by the Schrödinger equation with an appropriate Coulomb-nuclear potential.

The corresponding modified partial wave radial Schrödinger equation having reduced mass μ and energy $E = \hbar^2k^2/2\mu$ and governed by the sum of Coulomb potential $U_C(r)$ and $U_n(r) = (\hbar^2/2\mu)\, V_n(r)$ is

$$\frac{d^2\phi_l(\eta,kr)}{dr^2}+[k^2-\frac{l(l+1)}{r^2}-V_c(r)-V_n(r)]\phi_l(\eta,kr)=0$$

and $V_c(r)$ assumed is due to a uniformly charged sphere of radius $R_c = r_c\, A_T^{1/3}$ and $\eta = Z_1Z_2e^2\mu/k\hbar^2$ is the Rutherford parameter. The procedure used in the present study is elaborated in [2,3] which is the S-matrix method of searching resonant states by determining the poles of S-matrix in lower complex *k*-plane having small negative imaginary part.

In Table 1 we summarize our numerical results and compare our results using SM method with the empirical formulae used in calculating half-lives [5,6] which we denote as F1 and F2. In all calculations the exact nuclear potentials were used. The results (both Q values and half lives) obtained using fixed r_c = 1.24f (indicated by * in the Table 1) reasonably agree with the experimental results. It is to be noted that the experimental *Q* values are quite reliable while the measurement of half lives are quite uncertain in this super heavy region. Since $\tau_{1/2}$ is very sensitive to potential, the agreement with the experiment can be further improved by fine tuning r_c, keeping it within the reasonable range of 1.2f -1.3f. These results are also listed in the same table along with the fitted values of r_c. The table shows that with this

one parameter variation both Q and $\tau_{1/2}$ reproduce the corresponding experimental values quite well over a wide range of α-decaying systems. Comparison of results obtained from parametrizing r_c and fixed r_c = 1.24f demonstrates that $log\,\tau_{1/2}$ is quite sensitive to r_c but both generate Q values fairly close to each other**.** This is not surprising because barrier transmission which is primarily responsible for $log\,\tau_{1/2}$ is quite sensitive to minor changes in the potential in the barrier region. The empirical formula F1 and F2 [5,6] are defined with respect to *Q(exp)*. These empirical results are also listed in Table 1 for comparison. The present calculations show that RMF based α-daughter nucleus potentials are quite useful in a more accurate study of decay parameters for a wide range of systems using SM method.

Table 1. Numerical results of α-decay data analysis. *Q(exp)* and $\tau_{1/2}$*(exp)* refer to the experimental Q value and the corresponding half width [4,5]. The results obtained by SM method are denoted by Q(SM) and $\tau_{1/2}$(SM) where r_c was used as a parameter. For comparison we also list the corresponding results obtained for a fixed r_c = 1.24f and these are indicated by Q(SM)* and $\tau_{1/2}$(SM)*. Similarly, $\tau_{1/2}^{F1}$ and $\tau_{1/2}^{F2}$ indicate the results obtained for empirical formulae F1 and F2.

Sl. No	A	Z	Q(SM)* (MeV)	Q(SM) (MeV)	Q(ex) (MeV)	$\tau_{1/2}$ (SM)*	$\tau_{1/2}$ (SM)	$\tau_{1/2}$ (exp)	$\tau_{1/2}^{F1}$	$\tau_{1/2}^{F2}$	r_c (f)
1	294	118	11.92	11.84	11.81	1.13ms	1.76ms	1.8ms	0.39ms	0.64ms	1.243
2	292	116	11.14	11.16	10.80	20ms	17.05ms	18ms	27ms	49ms	1.24
3	291	116	11.29	11.12	10.89	2.25ms	6.3ms	6.3ms	89ms	336.4ms	1.247
4	290	116	11.91	11.21	11.00	0.29ms	14.20ms	15ms	8.9ms	15.2ms	1.267
5	289	114	10.87	10.10	9.96	20.2ms	2.66s	2.7s	6.1s	26.7s	1.269
6	288	114	10.92	10.31	10.09	17.49ms	0.79s	0.8s	0.52s	0.98s	1.263
7	287	114	10.15	10.33	10.16	1.81s	0.53s	0.51s	1.79s	7.24s	1.233
8	286	114	10.36	10.54	10.35	0.52s	0.157s	0.16s	0.11s	0.19s	1.233
9	284	113	10.29	10.24	10.15	0.36s	0.50s	0.48s	2.4s	4.13s	1.242
10	283	112	9.85	9.82	9.67	2.75s	3.28s	4.0s	9.6s	41.3s	1.241
11	280	111	9.87	9.8	9.87	1.19s	1.83s	3.6s	3.1s	5.70s	1.242
12	279	111	10.08	10.18	10.52	310ms	157ms	170ms	10.9ms	45.3ms	1.236
13	273	110	10.29	11.05	11.25	37.5ms	0.40ms	0.11ms	0.11ms	0.39ms	1.21
14	276	109	9.58	9.78	9.85	1.59s	0.42s	0.72s	0.65s	1.44s	1.232
15	275	109	9.7	10.36	10.48	0.73s	10.24ms	9.7ms	3.2ms	13.7ms	1.214
16	266	109	10.62	10.81	10.99	3.22ms	1.02ms	4.78ms	0.69ms	1.63ms	1.232
17	269	108	9.79	9.42	9.37	0.19s	2.48s	19.49s	0.68s	2.52s	1.255
18	267	108	10.06	9.88	10.03	37.97ms	117.0ms	74.1ms	32.9ms	112.5ms	1.247
19	272	107	9.09	9.72	9.15	9.83s	0.11s	9.8s	17.6s	33.8s	1.214
20	262	107	10.14	10.98	10.37	13ms	0.092ms	4.17ms	9.51ms	20.5ms	1.206
21	271	106	8.77	8.94	8.65	266s	12.23s	144s	108s	516s	1.233

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