The excited 2^+ state of ^{12}C in α cluster model

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Introduction

The analysis of ^{12}C in the α cluster model is considered to be an unsolved problem due to the inability to construct the trial wave function with correct asymptotic behavior for the unbound $\alpha\alpha$ pair. Despite this limitation, it has been analyzed in the Faddeev approach [1]. In our earlier work [2] we have also analyzed the ground state properties of ^{12}C in the α cluster model but using variational Monte Carlo (VMC) method. For the $\alpha\alpha$ pair, Ali and Bodmer potential [3] modified by Fedorov and Jensen [1] has been employed. We have successfully explained the ground state energy and rms radius consistent with the experimental data and the other approaches [1, 4]. The success of VMC for the ground state properties of ^{12}C has prompted us to extend it to analyze the excited 2^+ state. This is for the first time, we are applying VMC to the 2^+ state and we report the results of our calculation.

Hamiltonian, potential models, wavefunction and energy calculation

The ^{12}C in the $\alpha\alpha\alpha$ model is treated as a three-body system and its Hamiltonian for the pairwise $\alpha\alpha$ interaction can be written as:

$$\begin{aligned} H &= \sum_{i=1}^{3} K_\alpha(i) + V^{(0)}_{\alpha\alpha}(r_{12}) \\ &\quad + V^{(0)}_{\alpha\alpha}(r_{13}) + V^{(2)}_{\alpha\alpha}(r_{23}) \\ &\quad + V_{\alpha\alpha\alpha}(r_{12}, r_{13}, r_{23}), \end{aligned} \tag{1}$$

where 1, 2, 3 indicate the three α particles and K_α is the kinetic energy operator for α particle. $V^{(l)}_{\alpha\alpha}$ denotes the two-body potential for the $\alpha\alpha$ pair in the relative angular momentum state l(=0 for the ground and 2 for the excited state) and has the form

TABLE I: Potential parameters of Ali and Bodmer [3] from fit to $\alpha\alpha$-scattering data.

Angular momentum l	$V^{(l)}_{rep}$ (MeV)	$\beta^{(l)}_{rep}$ (fm)	$V^{(l)}_{att}$ (MeV)	$\beta^{(l)}_{att}$ (fm)
0	125.0	1.53	30.18	2.85
2	20.0	1.53	30.18	2.85

$$\begin{aligned} V^{(l)}_{\alpha\alpha}(r) &= V^{(l)}_{rep} \exp\Big[-\Big(\frac{r}{\beta^{(l)}_{rep}}\Big)^2\Big] \\ &\quad - V^{(l)}_{att} \exp\Big[-\Big(\frac{r}{\beta^{(l)}_{att}}\Big)^2\Big] \\ &\quad + V_{Coul}(r), \end{aligned} \tag{2}$$

where $V^{(l)}_i$ and $\beta^{(l)}_i$ are the strength and range parameters in the relative l state, respectively, for $i = rep(att)$. The parameters are listed in Table I. $V_{Coul}(r)$ is the finite size Coulomb $\alpha\alpha$ potential and $V_{\alpha\alpha\alpha}$, the three-body $\alpha\alpha\alpha$ Gaussian shape potential is given by

$$V_{\alpha\alpha\alpha} = W_3 \exp\big(-(r^2_{12} + r^2_{13} + r^2_{23})/\alpha^2\big), \tag{3}$$

with the strength $W_3 < 0$ and the range parameter α. The strength $W_3(= -16.0$ MeV) and range $\alpha(= 7.7$ fm) in Eq. (3) are adjusted so as a good fit to the experimental values of the binding energy and rms radius of the ^{12}C nucleus is obtained.

The trial wavefunction for a given system ^{12}C in the state (J, J_z) is the product of two-body correlation functions $f^{(l)}_{\alpha\alpha}$ in the relative angular momentum l state and the function $\eta^J_{J_z}$:

$$\begin{aligned} \Psi(J, J_z) &= f^{(0)}_{\alpha\alpha}(r_{12}) f^{(0)}_{\alpha\alpha}(r_{13}) f^{(2)}_{\alpha\alpha}(r_{23}) \\ &\quad \times \eta^J_{J_z}, \end{aligned} \tag{4}$$

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TABLE II: The calculated energies of ground and excited 2^+ states of ^{12}C are listed in column four for the states given in third column. Energies in bold face are from analyzes [1, 4, 5] (Experimental energy ^{12}C (g.s.) $E_B = -7.26$ MeV, ^{12}C $(2^+) = -2.84$ MeV.

	System	States	-E_B (MeV)
Our earlier work Ref. [2]	^{12}C	0^+	7.17
Ref. [1]			**6.81**
Ref. [4]			**7.26**
Present work		2^+	4.29
Ref. [5]			**4.99**

where (J, J_z)=$(2^+,\ J_z)$. The function η is obtained by coupling the spherical harmonic $Y_{2,m}(\Omega_{\alpha_i\alpha_j})$ and the appropriate spin function $\chi_{0,0}$. The correlation functions $f^{(l)}_{\alpha\alpha}(r)$, that have the asymptotic behavior $f_{hh}(r)\sim r^{-\nu_{hh}}$ $\exp(-\kappa_{hh}r)$, as usual, are obtained from the solution of the Schrödinger-type equation for the relative angular momentum state l of the $\alpha\alpha$ pair.

Energy calculation, results and discussion

The energy $-B$ for the system ^{12}C in the cluster model in the state (J, J_z) can be written as

$-B(J, J_z) =$

$$\frac{\langle\Psi(J, J_z)|H|\Psi(J, J_z)\rangle}{\langle\Psi(J, J_z)|\Psi(J, J_z)\rangle}. \qquad (5)$$

The VMC energy calculations were made for 100 000 points and statistical error is of the order of much less than 1% and is, therfore, not quoted in the table. The integrations were made upto 20 fm. The energy is minimized with respect to variational parameters using standard minimization code.

The results for the energy of excited 2^+ state of ^{12}C have been listed in Table II along with the result from the analysis [5] for comparison. In table we have also quoted the results from earlier analyzes [1, 2, 4] for the ground state of ^{12}C. From the table we note that our VMC energy for the 2^+ state is close to the correlated Gaussian basis calculation [5] but quite way off from the experimental value. In view of the satisfactory explanation of the excited state of ^{12}C we intend to extend this calculation to the excited degenerate doublet $(3^+/2, 5^+/2)$ of $^{13}_{\Lambda}$C.

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