Prediction for the ground state energy of ${}^{14}_{\Lambda\Lambda}$C in the cluster model

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Introduction

In the past, Shoeb and co-workers [1, 2], in the variational Monte Carlo (VMC) framework, have successfully explained the energies of the ground and excited states of ${}^{9}_{\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be in the α cluster model. The success is attributed to the proposed phenomenological dispersive [1] three-body $\Lambda\alpha\alpha$ cluster force inspired from the microscopic analysis of Bodmer *et al.* [3]. Recently, we have analyzed the ground state energies of ^{12}C and ${}^{13}_{\Lambda}$C in the $\alpha\alpha\alpha$ and $\Lambda\alpha\alpha\alpha$ models, respectively, alongwith the other α cluster strange and multistrange hypernuclei [4]. The phenomenological dispersive [1] three-body $\Lambda\alpha\alpha$ and $\alpha\alpha\alpha$ forces along with the two-body $\alpha\alpha$ and $\Lambda\alpha$ potentials explained the ground state energies. The VMC calculation of the energy [5] of the excited 2^+ state of ^{12}C in the α cluster model is found to be consistent with the observed value. Looking at the success of VMC calculation in explaining the ground and excited state properties of ^{12}C and ground state of ${}^{13}_{\Lambda}$C, we have extended it to predict the energy of ${}^{14}_{\Lambda\Lambda}$C in view of the ongoing programme to produce $S = -2$ hypernuclei at J-PARC.

Hamiltonian, wavefunction and energy calculation

The Hamiltonian for the five-body system ${}^{14}_{\Lambda\Lambda}$C in the $\Lambda\Lambda\alpha\alpha\alpha$ model takes the form:

$$\begin{aligned} H_{\Lambda\Lambda} \;=\; & \sum_{i=1}^{2} K_{\Lambda}(i) + \sum_{i=3}^{5} K_{\alpha}(i) + V_{\Lambda\Lambda}(r_{12}) \\ & + \sum_{i=1}^{2}\sum_{j=3}^{5} V_{\Lambda\alpha}(r_{ij}) \\ & + \sum_{i<j}\;{}_{i=3,4,j=4,5} V_{\alpha\alpha}(r_{ij}) \\ & + \sum_{i=1}^{2}\sum_{j<k}^{5}{}_{j=3} V_{\Lambda\alpha\alpha}(r_{ij}, r_{ik}) \\ & + V_{\alpha\alpha\alpha}(r_{34}, r_{35}, r_{45}), \end{aligned} \quad (1)$$

where indices (1,2), (3,4,5) label Λs, and α particles, respectively, K_Y is the kinetic energy operator for the particle $Y(=\Lambda, \alpha)$, V_{hh} denotes the potential for the pair of particles hh ($= \Lambda\Lambda, \Lambda\alpha, \alpha\alpha$), r_{kl} is the inter-particle separation for the pair having indices k and l. Here we have considered that all the hh pairs in a given system interact in the relative angular momentum s-state and $V_{\Lambda\alpha\alpha}$ is the phenomenological dispersive three-body $\Lambda\alpha\alpha$ potential and has a simple form

$$V_{\Lambda\alpha\alpha} = W_0 f(r_{\Lambda\alpha_1}) f(r_{\Lambda\alpha_2}), \quad (2)$$

where the strength $W_0 > 0$ gives repulsion. The radial factor $f(r)$ is of Yukawa form: $f(r) = \exp(-ar)/ar$, where a is the range parameter. The $V_{\Lambda\alpha\alpha}$ ($W_0 = 17.0$ MeV and $a = 0.5$ fm^{-1} for the case of two-body $\Lambda\alpha$ Isle potential) is constrained to fit the B_{Λ}, Λ-binding energy [1] of the ${}^{9}_{\Lambda}$Be in the three-body $\Lambda\alpha\alpha$ model. The inclusion of the dispersive $\Lambda\alpha\alpha$ force [1, 2] gives a good account of the ground and excited states of ${}^{9}_{\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be, as stated in the preceding section. The expressions for $V_{\alpha\alpha\alpha}$, three-body $\alpha\alpha\alpha$ Gaussian shape potential and for the two-body $V_{\alpha\alpha}$ potential in the

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relative angular momentum s-state have been given in our other paper [5].

The phenomenological s-state $\Lambda\alpha$ potential of Isle form, the $\Lambda\Lambda$ potential in the singlet state of the three-range Gaussian form adjusted to fit the ground state binding energy (7.26 MeV) of ${}^{6}_{\Lambda\Lambda}$He and $\alpha\alpha$ potential of Ali and Bodmer [6] are taken from our earlier work [1, 2].

The trial wavefunction for the system ${}^{14}_{\Lambda\Lambda}$C is the product of two-body correlation functions f_{hh} for the pair of particles, hh and the appropriate spin function ζ_0^0 and has the form:

$$\Psi_{\Lambda\Lambda} = \left[\prod_{i=1}^{2}\prod_{j=3}^{5} f_{\Lambda\alpha}(r_{ij})\right] f_{\Lambda\Lambda}(r_{12}) \times \left[\prod_{i<j_{i=3,4,j=4,5}} f_{\alpha\alpha}(r_{ij})\right]\zeta_0^0. \quad (3)$$

The correlation functions $f_{hh}(r)$ have the asymptotic behavior $f_{hh}(r)\sim r^{-\nu_{hh}}\exp(-\kappa_{hh}r)$ and are obtained from the solution of the Schrödinger-type equation.

The total energy E_B for the system ${}^{14}_{\Lambda\Lambda}$C in the cluster model for the trial wavefunction Eq. (3) is evaluated using the following relation:

$$E_B({}^{14}_{\Lambda\Lambda}C) = \frac{\langle\Psi_{\Lambda\Lambda}|H_{\Lambda\Lambda}|\Psi_{\Lambda\Lambda}\rangle}{\langle\Psi_{\Lambda\Lambda}|\Psi_{\Lambda\Lambda}\rangle}. \quad (4)$$

The VMC estimates of the energy were made for 100 000 points. A distance of 20 fm was chosen for the integration and the energy was optimized with respect to variational parameters using standard code.

Results and Discussion

The ground state energy of ^{12}C which we have calculated earlier [4] in the VMC framework in the α cluster model is 7.17 MeV, quite consistent with the experimental value 7.26 MeV. The results of variational calculation for ${}^{14}_{\Lambda\Lambda}$C are as under;

Energy $-E_B$ = 31.29 ± 0.0002 MeV with break up: total kinectic energy = 30.58 MeV, total potential enegy = −61.74 MeV, three-body dispersive energy $\langle V_{\Lambda\alpha\alpha}\rangle$ = 8.72 MeV, three-body cluster energy $\langle V_{\alpha\alpha\alpha}\rangle$ = −8.85 MeV. The root mean square radius between $\alpha\alpha$, $\mathbf{R}_{\alpha\alpha}$ = 3.48 fm; between $\Lambda\Lambda$, $\mathbf{R}_{\mathbf{\Lambda\Lambda}}$ = 3.22 fm; between c.m. of 3-alpha and Λ, $\mathbf{R}_{(\mathbf{3}\alpha)\mathbf{\Lambda}}$ = 2.36 fm; between Λ and α, $\mathbf{R}_{\mathbf{\Lambda}\alpha}$ = 3.09 fm; between c.m. of 3-alpha and α, $\mathbf{R}_{(\mathbf{3}\alpha)\alpha}$ = 2.01 fm; between c.m. of 3-alpha and $\Lambda\Lambda$, $\mathbf{R}_{(\mathbf{3}\alpha)(\mathbf{\Lambda\Lambda})}$ = 1.72 fm.

The predicted $B_{\Lambda\Lambda}$ ($= E_B({}^{12}\mathrm{C})$ $-E_B({}^{14}_{\Lambda\Lambda}\mathrm{C})$) for ${}^{14}_{\Lambda\Lambda}$C turns out to be 24.12 MeV.

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