Prediction for the ground state energy of $^{14}_{\Lambda\Lambda}C$ in the cluster model

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Introduction

In the past, Shoeb and co-workers [1, 2], in the variational Monte Carlo (VMC) framework, have successfully explained the energies of the ground and excited states of $^{9}_{\Lambda}Be$ and $^{10}_{\Lambda\Lambda}Be$ in the $\alpha$ cluster model. The success is attributed to the proposed phenomenological dispersive [1] three-body $\Lambda\alpha\alpha$ cluster force inspired from the microscopic analysis of Bodmer et al. [3]. Recently, we have analyzed the ground state energies of $^{12}_{\Lambda}C$ and $^{13}_{\Lambda}C$ in the $\alpha\alpha\alpha$ and $\Lambda\alpha\alpha$ models, respectively, along with the other $\alpha$ cluster strange and multistrange hypernuclei [4]. The phenomenological dispersive [1] three-body $\Lambda\alpha\alpha$ and $\alpha\alpha\alpha$ forces along with the two-body $\alpha\alpha$ and $\Lambda\alpha$ potentials explained the ground state energies. The VMC calculation of the energy [5] of the excited $2^+$ state of $^{12}_{\Lambda}C$ in the $\alpha$ cluster model is found to be consistent with the observed value. Looking at the success of VMC calculation in explaining the ground and excited state properties of $^{12}_{\Lambda}C$ and ground state of $^{13}_{\Lambda}C$, we have extended it to predict the energy of $^{14}_{\Lambda\Lambda}C$ in view of the ongoing programme to produce $S=-2$ hypernuclei at J-PARC.

Hamiltonian, wavefunction and energy calculation

The Hamiltonian for the five-body system $^{14}_{\Lambda\Lambda}C$ in the $\Lambda\Lambda\alpha\alpha\alpha$ model takes the form:

$$H_{\Lambda\Lambda} = \sum_{i=1}^{2} K_\Lambda(i) + \sum_{i=3}^{5} K_\alpha(i) + V_{\Lambda\Lambda}(r_{12})$$

$$+ \sum_{i=1}^{2} \sum_{j=3}^{5} V_{\Lambda\alpha}(r_{ij})$$

$$+ \sum_{i<j=3,4, j=4,5} V_{\alpha\alpha}(r_{ij})$$

$$+ \sum_{i=1}^{2} \sum_{j<k=3}^{5} V_{\Lambda\alpha\alpha}(r_{ij}, r_{ik})$$

$$+ V_{\alpha\alpha\alpha}(r_{34}, r_{35}, r_{45}),$$

where indices (1,2), (3,4,5) label $\Lambda$s, and $\alpha$ particles, respectively, $K_Y$ is the kinetic energy operator for the particle $Y(=\Lambda, \alpha)$, $V_{hh}$ denotes the potential for the pair of particles $hh$ ($=\Lambda\Lambda, \Lambda\alpha, \alpha\alpha$), $r_{kl}$ is the inter-particle separation for the pair having indices $k$ and $l$. Here we have considered that all the $hh$ pairs in a given system interact in the relative angular momentum $s$-state and $V_{\Lambda\alpha\alpha}$ is the phenomenological dispersive three-body $\Lambda\alpha\alpha$ potential and has a simple form

$$V_{\Lambda\alpha\alpha} = W_0 f(r_{\Lambda\alpha_1}) f(r_{\Lambda\alpha_2}),$$

where the strength $W_0 > 0$ gives repulsion. The radial factor $f(r)$ is of Yukawa form: $f(r) = \exp(-ar)/ar$, where $a$ is the range parameter. The $V_{\Lambda\alpha\alpha}$ ($W_0 = 17.0$ MeV and $a = 0.5$ fm$^{-1}$ for the case of two-body $\Lambda\alpha$ Iske potential) is constrained to fit the $B_\Lambda$, $\Lambda$-binding energy [1] of the $^{9}_{\Lambda}Be$ in the three-body $\Lambda\alpha\alpha$ model. The inclusion of the dispersive $\Lambda\alpha\alpha$ force [1, 2] gives a good account of the ground and excited states of $^{9}_{\Lambda}Be$ and $^{10}_{\Lambda\Lambda}Be$, as stated in the preceding section. The expressions for $V_{\alpha\alpha\alpha}$, three-body $\alpha\alpha\alpha$ Gaussian shape potential and for the two-body $V_{\alpha\alpha}$ potential in the

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relative angular momentum $s$-state have been given in our other paper [5].

The phenomenological $s$-state $\Lambda\alpha$ potential of Isle form, the $\Lambda\Lambda$ potential in the singlet state of the three-range Gaussian form adjusted to fit the ground state binding energy (7.26 MeV) of $^6\Lambda\Lambda$He and $\alpha\alpha$ potential of Ali and Bodmer [6] are taken from our earlier work [1, 2].

The trial wavefunction for the system $^{14}\Lambda\Lambda C$ is the product of two-body correlation functions $f_{hh}$ for the pair of particles, $hh$ and the appropriate spin function $\zeta_0^0$ and has the form:

$$\Psi_{\Lambda\Lambda} = \left[ \prod_{i=1}^{2} \prod_{j=3}^{5} f_{\Lambda\alpha}(r_{ij}) \right] f_{\Lambda\Lambda}(r_{12})$$

$$\times \left[ \prod_{i<j=3,4, j=4,5} f_{\alpha\alpha}(r_{ij}) \right] \zeta_0^0.$$  

The correlation functions $f_{hh}(r)$ have the asymptotic behavior $f_{hh}(r) \sim r^{-\nu_{hh}} \exp(-\kappa_{hh}r)$ and are obtained from the solution of the Schrödinger-type equation.

The total energy $E_B$ for the system $^{14}\Lambda\Lambda C$ in the cluster model for the trial wavefunction Eq. (3) is evaluated using the following relation:

$$E_B(\Lambda\Lambda C) = \frac{\langle \Psi_{\Lambda\Lambda} | H_{\Lambda\Lambda} | \Psi_{\Lambda\Lambda} \rangle}{\langle \Psi_{\Lambda\Lambda} | \Psi_{\Lambda\Lambda} \rangle}.$$  

The VMC estimates of the energy were made for 100 000 points. A distance of 20 fm was chosen for the integration and the energy was optimized with respect to variational parameters using standard code.

**Results and Discussion**

The ground state energy of $^{12}\alpha C$ which we have calculated earlier [4] in the VMC framework in the $\alpha$ cluster model is 7.17 MeV, quite consistent with the experimental value 7.26 MeV. The results of variational calculation for $^{14}\Lambda\Lambda C$ are as under:

Energy $-E_B = 31.29 \pm 0.0002$ MeV with break up: total kinetic energy = 30.58 MeV, total potential energy = $-61.74$ MeV, three-body dispersive energy $\langle V_{\Lambda\alpha\alpha} \rangle = 8.72$ MeV, three-body cluster energy $\langle V_{\alpha\alpha\alpha} \rangle = -8.85$ MeV. The root mean square radius between $\alpha\alpha$, $R_{\alpha\alpha} = 3.48$ fm; between $\Lambda\Lambda$, $R_{\Lambda\Lambda} = 3.22$ fm; between c.m. of 3-alpha and $\Lambda$, $R_{\alpha\alpha\alpha} = 2.36$ fm; between $\Lambda$ and $\alpha$, $R_{\Lambda\alpha} = 3.09$ fm; between c.m. of 3-alpha and $\alpha$, $R_{\alpha\alpha\alpha} = 2.01$ fm; between c.m. of 3-alpha and $\Lambda\Lambda$, $R_{\alpha\alpha\alpha}(\Lambda\Lambda) = 1.72$ fm.

The predicted $B_{\Lambda\Lambda}$ ($= E_B(\Lambda\Lambda C) - E_B(\Lambda\Lambda C)$) for $^{14}\Lambda\Lambda C$ turns out to be 24.12 MeV.

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**References**


