Aharonov-Bohm Effect in Nuclei

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Introduction

The effect of vector potential on the phase of a quantum wavefunction was brought out fifty years ago, and is termed as Aharonov-Bohm (AB) effect [1]. The effect is described in terms of the shift in the interference pattern created by a charged particle moving outside a solenoid or flux line. In a remarkable experiment, this effect was experimentally verified in precise quantitative terms [2]. There is an enormous literature on basic questions and effects related to this effect (see e.g. [3]). Yang has unified electromagnetic and gravitational AB effects [4]. As quoted in a recent article [5], "Aharonov stresses that the arguments that led o the prediction of the various electromagnetic AB effects apply well to any gauge-invariant quantum theory. In the standard model of particle physics, the strong and weak nuclear interactions are also described by gauge-invariant theories." This work presents AB effect presented in the ground state of deuteron, thus it might well be called "nuclear AB effect".

The work is based on the study of ground state wavefunction of deuteron to $O(\hbar^2)$ which, in principle, can be extended to all orders [6]. Beginning with the coupled equations describing the $^3S_1$ state, $u(r)$ and $^3D_1$ state, $w(r)$ [7], it was shown that Hamiltonian for the deuteron may be written as

$$\mathcal{H} = \frac{1}{M}(p_r\mathbf{I} - \hbar\mathbf{A}(r))^2 + \mathbf{v} \qquad (1)$$

where p_r, M, $\mathbf{I}$, $\mathbf{v}$ are respectively radial momentum, reduced mass pf the proton-neutron system, identity and potential matrix. The potential matrix consists of two potential energy surfaces, one binding (v_+) and the other scattering (v_-), details may be found in [6]. The vector potential $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{id\theta}{2dr} \\ \frac{-id\theta}{2dr} & 0 \end{bmatrix} \qquad (2)$$

This is similar to the vector potential that occurs in the context of electromagnetic interactions. Let us note that in the case of deuteron, it is appearing due to the tensor interaction which, in turn, is responsible for the coupling of the two states. Let us also observe that this implies that the radial wavefunction of deuteron is not a scalar but a two-component column vector.

For the deuteron, the Hamiltonian up to $O(\hbar^2)$ consists of two expressions, $h^{(\pm)}$, given by

$$h^{(\pm)} = \left(-\frac{\hbar^2}{M}\right)\left[1 \pm \frac{\hbar^2}{M(v_+(r) - v_-(r))} \theta'^2(r)\right]\frac{d^2}{dr^2} + v_\pm(r) + \frac{\hbar^2}{8M}\theta'^2(r) \qquad (3)$$

where θ' is the first derivative w.r.t. r, and the effective masses, $M_\pm$ are

$$M_\pm(r) = \frac{M}{1 \pm \frac{\hbar^2}{M}\frac{1}{v_+ - v_-}\left(\frac{d\theta}{dr}\right)^2}. \qquad (4)$$

The semiclassical wave equations are

$$h^{(\pm)}\psi^\pm = E\psi^\pm \qquad (5)$$

where

$$\psi^+ = u(r)\cos\frac{\theta(r)}{2} - w(r)\sin\frac{\theta(r)}{2},$$
$$\psi^- = u(r)\sin\frac{\theta(r)}{2} + w(r)\cos\frac{\theta(r)}{2}. \qquad (6)$$

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Mixing angle $\theta(r)$ is given in terms of central and tensor potentials by

$$\tan\theta = \frac{\sqrt{8}V_T(r)}{\frac{3\hbar^2}{Mr^2} - V_T(r)}. \quad (7)$$

AB effect in perturbation theory

Let us consider (1) again and re-write it keeping the first two dominant terms in terms of $\hbar$:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + O(\hbar^2) \quad (8)$$

where the dominant part in a power series of $\hbar$ is $\mathcal{H}_0 = \frac{p_r^2}{M}\mathbf{I} + \mathbf{v}$ and the next term at $O(\hbar)$ is $\mathcal{H}_1 = \frac{\hbar}{M}(\mathbf{A}p_r + p_r\mathbf{A})$. Explicitly, $\mathcal{H}_1$ is

$$\mathcal{H}_1 = \begin{bmatrix} 0 & \frac{d\theta}{dr}\frac{\partial}{\partial r} + \frac{d^2\theta}{dr^2} \\ -\frac{d\theta}{dr}\frac{\partial}{\partial r} - \frac{d^2\theta}{dr^2} & 0 \end{bmatrix}. \quad (9)$$

Considering $\psi^{\pm}$ as eigenstates of $\mathcal{H}_0$, the interaction $\mathcal{H}_1$ induces transitions between states, to first order in $\hbar$. Expectation value of this operator in the state, $\mathbf{\Psi} = (\psi^+, \psi^-)^T$ (superscript, T stands for transpose) can be shown to be

$$\mathbf{\Psi}^T\mathcal{H}_1\mathbf{\Psi} = \frac{\hbar^2\theta'}{2M}\left(\psi^+\frac{\partial\psi^-}{\partial r} - \psi^-\frac{\partial\psi^+}{\partial r}\right). (10)$$

This is non-zero because the quantity in brackets on the RHS is simply the Wronskian of linearly independent radial solutions ψ^+ and ψ^-. The transition probability amplitude for the state to flip is also non-zero:

$$\begin{bmatrix}\psi^- & \psi^+\end{bmatrix}\mathcal{H}_1\begin{bmatrix}\psi^+ \\ \psi^-\end{bmatrix} = \frac{\hbar^2\theta'}{4M}.$$
$$.\frac{\partial}{\partial r}[(w^2 - u^2)\cos 2\theta + 2uw\sin 2\theta]. (11)$$

Thus we reach the conclusion that in the ground state of deuteron, there are continuous transitions taking place between the two "mass states", effected by the vector potential coupled to momentum. Remember that no energy changes are involved in these transitions as both $\psi^{\pm}$ correspond to the same ground state energy of deuteron. Transition rate will be given by the square of the modulus of (11).

Nearly thirty years ago, it was shown [8] that AB effect occurs in a perturbation calculation where the electrons are scattered from a magnetic flux located on the z-axis, and that it occurred only when two eigenfunctions of L_z having eignvalues of opposite sign are mixed - these different combinations corresponded to different paths. In this paper, we are in a similar situation in a very subtle manner - the two paths correspond to the two combinations of angular momentum states mixed due to tensor interaction, and, their being at the same energy is due to the fact that they both correspond to ground state. In the absence of vector potential, there would be no transitions, no AB effect, and also no tensor potential, and hence no stable ground state combining $l = 0$ and $l = 2$ states, which means no stable deuteron.

References

[1] Y. Aharonov, D. Bohm, Phys. Rev. A **115**, 485 (1959).

[2] A. Tonomura et al., Phys. Rev. Lett. **48**, 1443 (1982).

[3] Y. Aharonov, T. Kaufherr, Phys. Rev. Lett. **92**, 070404 (2004).

[4] C. N. Yang, Phys. Rev. **33**, 445 (1974).

[5] H. Batelaan, A. Tonomura, Phys. Today, p. 38 (September 2009); I thank K. Mahata for bringing this reference to my attention.

[6] S. R. Jain, J. Phys. G **30**, 157 (2004).

[7] W. Rarita, J. Schwinger, Phys. Rev. **59**, 436 (1941).

[8] K. M. Purcell, W. C. Henneberger, Am. J. Phys. **46**, 1255 (1978).