Short Range $\alpha$-$\alpha$ Repulsion and FR-DWIA Analysis of the ($\alpha$, 2$\alpha$) reaction on $^9$Be and $^{20}$Ne

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The $^9$Be and $^{20}$Ne nuclei are supposed to be highly $\alpha$-clustered because while $^9$Be is a Borromean nucleus the $^{20}$Ne nucleus is having 4-nucleons outside the closed shell $^{16}$O nucleus. The same is also anticipated from the small $\alpha$-separation energies for these two nuclei which are 2.4672 MeV and 4.7316 MeV respectively, in comparison the values for $^{16}$O and $^{12}$C nuclei are 7.1622 MeV, 7.367 MeV respectively. Therefore the $\alpha$-bound wave functions for these nuclei are expected to extend to larger $\alpha$-residual nucleus separations. Which is reflected in higher ($\alpha$, 2$\alpha$) reaction cross section on these nuclei which are indeed seen to be higher than the ones from $^{12}$C and $^{16}$O nuclei. The symmetric coplanar peak ($\alpha$, 2$\alpha$) cross section at 140 MeV on $^9$Be and $^{20}$Ne are seen to be 0.1 and 0.035 as compared to 0.018 and 0.01 on $^{12}$C and $^{16}$O respectively. Earlier the ($\alpha$, 2$\alpha$) reactions on $^{12}$C and $^{16}$O at 140 MeV analyzed with the conventional ZR-DWIA formalism indicated orders of magnitude lower cross sections. Finite range-DWIA calculations using a short range repulsive $\alpha$$\alpha$ core effective interaction however brought a good agreement between theory and experiment. In order to verify the trend seen in $^{12}$C and $^{16}$O the theory should repeat itself in $^9$Be and $^{20}$Ne also. We therefore performed these FR-DWIA calculations for ($\alpha$, 2$\alpha$) reactions on $^9$Be and $^{20}$Ne. Using the transition amplitude, $T_{\alpha L \Lambda}$ for A($\alpha$, 2$\alpha$)B reaction in the FR-DWIA formalism the cross section can be written as,

$$\frac{d^3\sigma_{LJ}}{d\Omega_1 d\Omega_2 dE_1} = F_{\text{kin}} \cdot S_{\alpha}^{LJ} \cdot \sum_{\Lambda} |T_{\alpha L \Lambda}(\vec{k}_f, \vec{k}_i)|^2$$

where $F_{\text{kin}}$ is a kinematic factor and $S_{\alpha}^{LJ}$ is the cluster spectroscopic factor. The conventional transition matrix element for the knockout reaction, $T_{\alpha L \Lambda}(\vec{k}_f, \vec{k}_i)$ is

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The distorted waves \( \chi_0^- \left( \vec{k}_{1A}, \vec{r}_{1B} \right) \chi_2^- \left( \vec{k}_{2B}, \vec{R}_{2B} \right) t_{12} \left( \vec{r}_{12} \right) \)

where, \( t_{12} \left( E, \vec{r} \right) = e^{-ikz} V \left( \vec{r} \right) \Psi_{12}^* \left( \vec{r} \right) \equiv \sum_{L=0,1,2,...} t_L(E, r) P_L(\vec{r}) \)

The distorted waves \( \chi_0, \chi_1 \) and \( \chi_2 \) are evaluated using the optical potentials for the \( A, A-1 \) and \( A-2 \) pairs. Finally all the relative coordinates are expressed in terms of \( \vec{r}_{12} (\equiv \vec{r}) \) and \( \vec{R}_{2B} (\equiv \vec{R}) \). While using the ZR-DWIA the transition matrix element, \( T_{f_i} \) was factorized into integrals over \( \vec{r} \) and \( \vec{R} \) separately. The same is not possible, when one uses the full finite range \( t_{12}(\vec{r}_{12}) \) [1], due to the presence of optical distortions. This is because in the FR-DWIA formalism the chosen relative coordinates \( \vec{r} \) and \( \vec{R} \) get coupled through the distorted waves \( \chi_0^{(-)}(\vec{k}_{1A}, \vec{r}_{1B}) \) and \( \chi_1^{(-)}(\vec{k}_{1A}, \vec{r}_{1B}) \).

For the evaluation of \( T_{f_i}^0 \) the distorted waves, \( \chi(\vec{k}, \vec{r}) \) and \( \varphi_{L^\omega}(\vec{R}) \) and \( t_{12}(\vec{r}) \) were evaluated on the mesh of the spherical polar coordinates, \( r, \theta, \phi \) and \( R, \Theta, \Phi \). The final result of \( T_{f_i} \) is obtained by doing a 6-dimensional integration over the mesh of \( \vec{r} \) and \( \vec{R} \) coordinates.

The results of the FR-DWIA computations for \( ^9\text{Be} \) and \( ^{20}\text{Ne} \), normalized to the data peak values, are presented in Figs.1-2. Although the shapes of the energy sharing distributions (\( \sigma_{(\alpha,2\alpha)}(E_1) \) vs \( E_1 \)) are not very satisfactory the curves obtained from the attractive, \( t_{\alpha\alpha}(A)(\vec{r}) \) are much closer to the data.

This arises because the \( t_{\alpha\alpha}(A)(\vec{r}) \)'s peak close to \( r=0 \), which simulates the zero range behavior and hence the results are similar to the ZR-DWIA results. The repulsive core, \( (R+A)^2 \) results are seen to be at much variance. This could arise due to the uncertainty in the choice of the repulsive core \( \alpha-\alpha \) potential parameters. Most important conclusion however, can be drawn by comparison (bold face entries) of the absolute peak cross section values from the FR-DWIA calculations with the data and the derived \( S_{\alpha} \)-values from theory in Table 1.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( E_\alpha ) (MeV)</th>
<th>( \sigma_{\alpha,2\alpha}(\text{Peak})/\text{Sr}^2\text{MeV} )</th>
<th>Expt.</th>
<th>( S_{\alpha} ) (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^9\text{Be}(\alpha,2\alpha)^7\text{He} )</td>
<td>197</td>
<td>575 ( \mu )b</td>
<td>26.4 ( \mu )b</td>
<td>6.3 ( \mu )b</td>
</tr>
<tr>
<td>(^{12}\text{C}(\alpha,2\alpha)^8\text{Be} )</td>
<td>200</td>
<td>19.9 ( \mu )b</td>
<td>552 ( \mu )b</td>
<td>380 ( \mu )b</td>
</tr>
<tr>
<td>(^{16}\text{O}(\alpha,2\alpha)^{12}\text{C} )</td>
<td>140</td>
<td>92 ( \mu )b</td>
<td>2.5 ( \mu )b</td>
<td>18.5 ( \mu )b</td>
</tr>
<tr>
<td>(^{20}\text{Ne}(\alpha,2\alpha)^{16}\text{O} )</td>
<td>140</td>
<td>73.6 ( \mu )b</td>
<td>1.9 ( \mu )b</td>
<td>32 ( \mu )b</td>
</tr>
</tbody>
</table>

This Table I: Comparison of \( (\alpha, 2\alpha) \) cross sections from FR-DWIA calculations and experimental data on \( ^9\text{Be}, ^{12}\text{C}, ^{16}\text{O} \) and \( ^{20}\text{Ne} \) at various energies and spectroscopic factors \( (S_{\alpha}) \) derived from the FR-DWIA calculations and theory. Comparison of Bold face entries is emphasized.