Short Range α-α Repulsion and FR-DWIA Analysis of the (α, 2α) reaction on ^{9}Be and ^{20}Ne

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The ^{9}Be and ^{20}Ne nuclei are supposed to be highly α-clustered because while ^{9}Be is a Borromean nucleus the ^{20}Ne nucleus is having 4-nucleons outside the closed shell ^{16}O nucleus. The same is also anticipated from the small α-separation energies for these two nuclei which are 2.4672 MeV and 4.7316 MeV respectively, in comparison the values for ^{16}O and ^{12}C nuclei are 7.1622 MeV, 7.367 MeV respectively. Therefore the α-bound wave functions for these nuclei are expected to extend to larger α-residual nucleus separations. Which is reflected in higher (α, 2α) reaction cross section on these nuclei which are indeed seen to be higher than the ones from ^{12}C and ^{16}O nuclei. The symmetric coplanar peak (α, 2α) cross section at 140 MeV on ^{9}Be and ^{20}Ne are seen to be 0.1 and 0.035 as compared to 0.018 and 0.01 on ^{12}C and ^{16}O respectively. Earlier the (α, 2α) reactions on ^{12}C and ^{16}O at 140 MeV analyzed with the conventional ZR-DWIA formalism indicated orders of magnitude lower cross sections. Finite range-DWIA calculations using a short range repulsive α-α core effective interaction however brought a good agreement between theory and experiment. In order to verify the trend seen in ^{12}C and ^{16}O the theory should repeat itself in ^{9}Be and ^{20}Ne also. We therefore performed these FR-DWIA calculations for (α, 2 α) reactions on ^{9}Be and ^{20}Ne. Using the transition amplitude, T_{fi} for A$(\alpha, 2\alpha)$B reaction in the FR-DWIA formalism the cross section can be written as,

$$\frac{d^3\sigma^{L,J}}{d\Omega_1 d\Omega_2 dE_1} = F_{kin} \cdot S_{\alpha}^{LJ} \cdot \sum_{\wedge} |T_{fi}^{\alpha L\wedge}(\vec{k}_f, \vec{k}_i)|^2$$

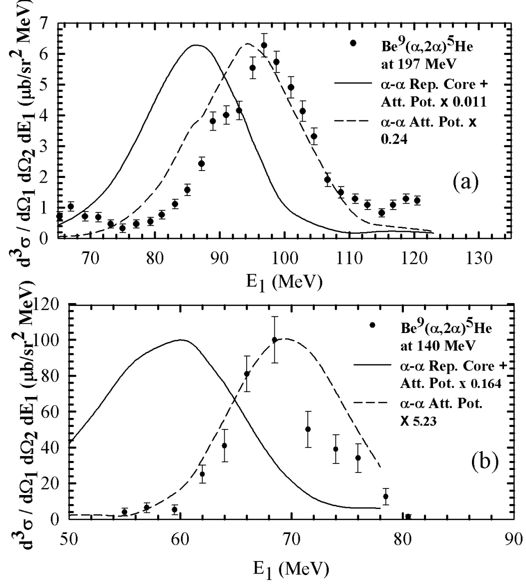

FIG. 1: Comparison of ^{9}Be$(\alpha, 2\alpha)$ data with the FR-DWIA calculations using α-α interaction which is purely attractive(A) and having a repulsive core (R+A), (a) for 197 MeV and (b) for 140 MeV.

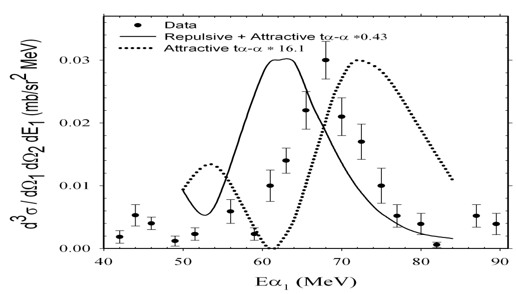

FIG. 2: Same as Fig.1 but for 140 MeV ^{20}Ne$(\alpha, 2\alpha)$ data.

where F_{kin} is a kinematic factor and S_{α}^{LJ} is the cluster spectroscopic factor. The conventional transition matrix element for the knockout reaction, $T_{fi}^{\alpha L\wedge}(\vec{k}_f, \vec{k}_i)$ is

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TABLE I: Comparison of (α, 2 α) cross sections from FR-DWIA calculations and experimental data on ^{9}Be, ^{12}C, ^{16}O and ^{20}Ne at various energies and spectroscopic factors (S_α) derived from the FR-DWIA calculations and theory. Comparison of Bold face entries is emphasized.

Reaction	E_α (MeV)	$\sigma_{\alpha,2\alpha}(Peak)/Sr^2$MeV (R+A)	(A)	Expt	S_α (R+A)	(A)	Theory
$^{9}Be(\alpha,2\alpha)^{5}He$	197	575 μb	**26.4** μb	**6.3** μb	0.011	**0.24**	**0.57**
	140	**609** μb	19.1 μb	**100** μb		**0.164**	5.23
$^{12}C(\alpha,2\alpha)^{8}Be$	200	19.9 μb	**552** nb	**380** nb	0.02	**0.7**	**0.55**, **0.29**
	140	**92** μb	2.5 μb	**18.5** μb		**0.2**	7.4
$^{16}O(\alpha,2\alpha)^{12}C$	140	**19.1** μb	0.51 μb	**10.5** μb	**0.55**	20.6	**0.23**, **0.3**
	90	**171** μb	14.3 μb	**68** μb	**0.4**	4.75	
$^{20}Ne(\alpha,2\alpha)^{16}O$	140	**73.6** μb	1.9 μb	**32** μb	**0.435**	16.1	**0.21**

$$= \int \chi_1^{(-)*}(\vec{k}_{1B},\vec{r}_{1B})\chi_2^{(-)*}(\vec{k}_{2B},\vec{R}_{2B})t_{12}(\vec{r}_{12}) \chi_0^{(+)}(\vec{k}_{1A},\vec{r}_{1A})\varphi_{L\wedge}(\vec{R}_{2B})d\vec{r}_{12}d\vec{R}_{2B}$$

where, $t_{12}^{+}(E,\vec{r}) = e^{-ikz}V(\vec{r})\Psi_{12}^{+}(\vec{r}) \equiv \sum_{L=0,1,2,...} t_L(E,r)P_L(\hat{r})$

The distorted waves χ_0, χ_1 and χ_2 are evaluated using the optical potentials for the α_1-A, α_1-B and α_2-B pairs. Finally all the relative coordinates are expressed in terms of $\vec{r}_{12}(\equiv \vec{r})$ and $\vec{R}_{2B}(\equiv \vec{R})$. While using the ZR-DWIA the transition matrix element, T_{fi} was factorized into integrals over $\vec{r}$ and $\vec{R}$ separately. The same is not possible, when one uses the full finite range $t_{12}(\vec{r}_{12})$ [1], due to the presence of optical distortions. This is because in the FR-DWIA formalism the chosen relative coordinates $\vec{r}$ and $\vec{R}$ get coupled through the distorted waves $\chi_0^{(+)}(\vec{k}_{1A},\vec{r}_{1A})$ and $\chi_1^{(-)*}(\vec{k}_{1B},\vec{r}_{1B})$.

For the evaluation of $T_{fi}^{\alpha,L,\wedge}$ the distorted waves, $\chi(\vec{k},\vec{r})$ and $\varphi_{L\wedge}(\vec{R})$ and $t_{12}(\vec{r})$ were evaluated on the mesh of the spherical polar coordinates, r, θ, ϕ and R, Θ, Φ. The final result of T_{fi} is obtained by doing a 6-dimensional integration over the mesh of $\vec{r}$ and $\vec{R}$ coordinates.

The results of the FR-DWIA computations for ^{9}Be and ^{20}Ne, normalized to the data peak values, are presented in Figs.1-2. Although the shapes of the energy sharing distributions ($\sigma_{(\alpha,2\alpha)}(E_1)$ vs E_1) are not very satisfactory the curves obtained from the attractive, $t_{\alpha\alpha(A)}(\vec{r})$ are much closer to the data. This arises because the $t_{\alpha\alpha(A)}(\vec{r})$'s peak close to r=0, which simulates the zero range behavior and hence the results are similar to the ZR-DWIA results. The repulsive core, (R+A) results are seen to be at much variance. This could arise due to the uncertainty in the choice of the repulsive core α-α potential parameters. Most important conclusion however, can be drawn by comparison (bold face entries) of the absolute peak cross section values from the FR-DWIA calculations with the data and the derived S_α-values from theory in Table 1.

Here it is seen that the absolute cross sections and S_α values for the $\sim$ 197-200 MeV $(\alpha,2\alpha)$ reactions on ^{9}Be and ^{12}C using the purely attractive $t_{\alpha\alpha(A)}(\vec{r})$ are in better agreement with data in comparison to that using $t_{\alpha\alpha(R+A)}(\vec{r})$ where the absolute cross sections are 20 to 35 times larger. For energies at and below $\sim$ 140 MeV,the peak cross sections close to the zero recoil momentum position (normalized to the data peak values) yield S_α-values much closer to the theoretical values when $t_{\alpha\alpha(R+A)}(\vec{r})$'s are employed. On the other hand, the S_α-values obtained from the $t_{\alpha\alpha(A)}(\vec{r})$'s are 10 to 90 times too large. It is thus concluded that ^{9}Be and ^{20}Ne also strengthen our earlier finding that for 140 MeV or below the (α, 2 α) results are nicely fitted with repulsive α-α core interaction.

References

[1] A. K. Jain and B. N. Joshi, Prog. of Theor. Phys **120**, 1193 (2008).