

## Fission of $^{246}\text{Bk}^*$ formed in $^{11}\text{B}+^{235}\text{U}$ and $^{14}\text{N}+^{232}\text{Th}$ reactions using the proximity potential in Wong formula

Manie Bansal and Raj K. Gupta

*Physics Department, Panjab University, Chandigarh-160014, INDIA*

### Introduction

The highly fissile compound nucleus (CN)  $^{246}\text{Bk}^*$ , formed in  $^{11}\text{B}+^{235}\text{U}$  and  $^{14}\text{N}+^{232}\text{Th}$  reactions at sub- and near-barrier energies [1], decays totally via fission whose cross-section is taken as the measure of fusion cross-section. This makes  $^{246}\text{Bk}^*$  an ideal case for studying the CN fusion-fission process in heavy mass nuclei formed in low energy heavy ion reactions. Furthermore, the measured fission fragment anisotropies in these experiments show the entrance channel effects for the anisotropy values of  $^{11}\text{B}+^{235}\text{U}$  being consistent, but that of  $^{14}\text{N}+^{232}\text{Th}$  anomalous, *w.r.t.* the statistical saddle-point model [1], but vice-versa in the dynamical cluster-decay model (DCM) calculation [2]. In other words, in contrast to experiments [1], in DCM [2] a non-CN, quasi-fission (qf) component seems to be present in the fusion cross-section for the  $^{11}\text{B}+^{235}\text{U}$  channel, rather than for the  $^{14}\text{N}+^{232}\text{Th}$  channel.

In this paper, we study for the first time the dynamics of these fission reactions on the basis of the Wong formula [3], which has so-far been used quite extensively for the dominantly fusion-evaporation and the capture (equivalently, quasi-fission) cross-sections. Our recent study [4] on some such specific reactions shows that, though the simple Wong formula based on angular momentum  $\ell=0$  barrier does not describe either the fusion-evaporation (in  $^{58}\text{Ni}+^{58}\text{Ni}$ ,  $^{64}\text{Ni}+^{64}\text{Ni}$  and  $^{100}\text{Mo}$ ) or capture cross-sections (in  $^{48}\text{Ca}+^{238}\text{U}$ ,  $^{244}\text{Pu}$  and  $^{248}\text{Cm}$ ), the  $\ell$ -summed Wong expression fits very well the capture cross-sections at all  $E_{c.m.}$ 's, but requires additional barrier modification effects to fit the fusion-evaporation cross-sections at sub-barrier energies. In the following, it will be interesting to see, what happens in the case of dominant fission reactions.

### The Model

According to Wong [3], the fusion cross-section, in terms of  $\ell$  partial waves, for two deformed and oriented nuclei (with orientation angles  $\theta_i$ ) lying in same plane (azimuthal angle  $\Phi=0$ ) and colliding with center-of-mass (c.m.) energy  $E_{c.m.}$ , is

$$\sigma(E_{c.m.}, \theta_i) = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) P_{\ell}(E_{c.m.}, \theta_i), \quad (1)$$

with  $k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}}$ , and  $\mu$  as the reduced mass. Here,  $P_{\ell}$  is the transmission coefficient for each  $\ell$  which describes the penetration of barrier  $V_{\ell}(R, E_{c.m.}, \theta_i)$ , determined in Hill and Wheeler approximation.

Instead of solving Eq. (1) explicitly, which require the complete  $\ell$ -dependent potentials  $V_{\ell}(R, E_{c.m.}, \theta_i)$ , Wong [3] carried out the  $\ell$ -summation in Eq. (1) approximately under the conditions of using only  $\ell=0$  quantities:

- (i)  $\hbar\omega_{\ell} \approx \hbar\omega_0$ , and
- (ii)  $V_B^{\ell} \approx V_B^0 + \frac{\hbar^2 \ell(\ell+1)}{2\mu R_B^0{}^2}$ ,

which means to assume also  $R_B^{\ell} \approx R_B^0$ . Using the above two approximations, and replacing the  $\ell$ -summation in Eq. (1) by an integral, gives on integration the Wong formula [3]

$$\sigma(E_{c.m.}, \theta_i) = \frac{R_B^0{}^2 \hbar\omega_0}{2E_{c.m.}} \ln \left[ 1 + \exp \left( \frac{2\pi}{\hbar\omega_0} (E_{c.m.} - V_B^0) \right) \right] \quad (2)$$

which on integrating over the orientation angles  $\theta_i$  gives the fusion cross-section

$$\sigma(E_{c.m.}) = \int_{\theta_i=0}^{\pi/2} \sigma(E_{c.m.}, \theta_i) \sin\theta_1 d\theta_1 \sin\theta_2 d\theta_2. \quad (3)$$

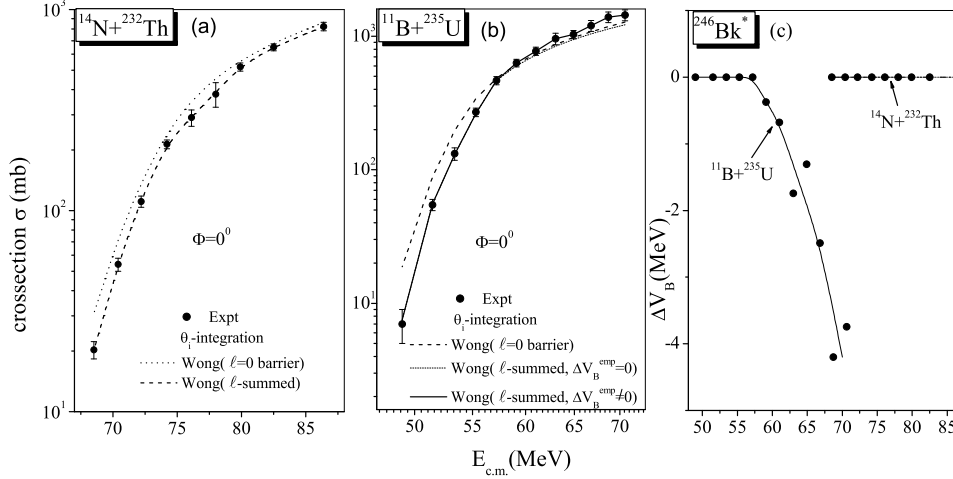


FIG. 1: Fission cross-section for (a)  $^{14}\text{N}+^{232}\text{Th}$  (b)  $^{11}\text{B}+^{235}\text{U}$  calculated for cases of  $\ell=0$  barrier based Wong formula, and  $\ell$ -summed Wong expression, with  $\Delta V_B^{emp}=0$  and best fitted  $\Delta V_B^{emp}$  at various  $E_{c.m.}$ , compared with experimental data. (c) The variation of  $\Delta V_B^{emp}$  with  $E_{c.m.}$ , for the best fit of data in (a) and (b) for both the channels. Here, the solid line is only for the guide of eye.

For an explicit summation over  $\ell$  in Eq. (1), the  $\ell$ -dependent interaction potential  $V_\ell(R)$ , is

$$V_\ell(R) = V_P(R, A_i, \beta_{\lambda i}, T, \theta_i) + \frac{\hbar^2 \ell(\ell+1)}{2\mu R^2} + V_C(R, Z_i, \beta_{\lambda i}, T, \theta_i), \quad (4)$$

with details of Coulomb and nuclear proximity potential for deformed, oriented nuclei given in [5]. The  $\ell$ -summation in Eq. (1) is then carried out for the  $\ell_{max}$  determined empirically. This procedure of explicit  $\ell$ -summation seems to work very well for the fission reaction  $^{14}\text{N}+^{232}\text{Th}$ , but not for  $^{11}\text{B}+^{235}\text{U}$  at higher energies (where the process of quasi-fission is prevalent [2]), and demands modification of the barrier, which could be carry out empirically [4] by either (i) keeping the curvature  $\hbar\omega_\ell$  same and modifying the barrier height  $V_B^\ell$ , obtained from Eq. (4), by  $\Delta V_B^{emp}$ , i.e., define

$$V_B^\ell(\text{modified}) = V_B^\ell + \Delta V_B^{emp},$$

or (ii) keep the barrier height  $V_B^\ell$  same and modify the curvature  $\hbar\omega_\ell$  as

$$\hbar\omega_\ell(\text{modified}) = \hbar\omega_\ell + \Delta\hbar\omega^{emp}.$$

Here, we use the method of modifying the barrier height.

## Calculations and results

Figs. 1 (a) and (b) show that the  $\ell=0$  barrier based Wong formula (2) does not fit either of the two fission reaction data, but the  $\ell$ -summed Wong expression (1) fits the  $^{14}\text{N}+^{232}\text{Th}$  data at all  $E_{c.m.}$ 's in Fig. 1(a), without any barrier lowering ( $\Delta V_B^{emp}=0$ ). On the other hand, Fig. 1(b) for  $^{11}\text{B}+^{235}\text{U}$  shows that barrier lowering is essential, but only at higher  $E_{c.m.}$ 's. The empirically fitted  $\Delta V_B^{emp}$  values are shown in Fig. 1(c). Based on the DCM calculation for the two reactions [2], the above result can be taken to say that, whereas  $^{14}\text{N}+^{232}\text{Th}$  is a pure CN reaction, a quasi-fission component exists in  $^{11}\text{B}+^{235}\text{U}$  reaction at higher energies.

## References

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