Semi-classical extended Thomas Fermi model and sudden- or frozen-density approximation used in the Wong formula

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Introduction

The semiclassical extended Thomas Fermi (ETF) method of Skyrme energy density formalism (SEDF) provides a convenient basis for the calculation of nucleus-nucleus interaction potential. Here, both the kinetic energy density τ and spin-orbit density $\vec{J}$ are functions of the nucleon density ρ_q, $q = n, p$. For the composite system, in ETF, the densities can be added either in sudden or frozen approximation [1]. Sudden-density contains the exchange terms due to anti-symmetrization whereas the frozen-density has no such effects in it. The exchange effects arise since, for the composite system, $\tau(\rho)$ and $\vec{J}(\rho)$ are expressed as functions of the ρ_i (i=1,2 for two nuclei), which, in turn, are the sums of their nucleon densities ($\rho_i = \rho_{in} + \rho_{ip}$), with $\rho = \rho_1 + \rho_2$. On the other hand, in frozen-density, the composite nucleus densities are simply the sums of the densities of two incoming nuclei.

In a recent paper [2], using Wong's approximate ℓ=0 barrier based formula [3] in ETF with sudden-densities, we were able to fit reasonably well (not exactly) the fusion-evaporation cross-sections for atleast the ^{64}Ni+^{64}Ni reaction with the barrier modified (lowered) by varying the half-density radius R_0 and surface thickness a_0 parameters of the two-parameter Fermi density (see Fig. 1(a)). An exactly similar fit is also obtained for the frozen-densities (also, see Fig. 1(a)), using another R_0 and a_0 parameter set. However, the same could not be achieved for, say, ^{58}Ni+^{58}Ni data. In fact, the barrier modification effects are also contained in the Wong's ℓ-summed expression, neglected in its ℓ=0 barrier based formula [3]. In this paper, we apply the ℓ-summed Wong expression, in both the sudden and frozen-density approximations, to the cases of ^{58,64}Ni+^{58,64}Ni reactions.

The semiclassical ETF model

The interaction potential in SEDF is

$$V_N(R) = \int \{H(\rho,\tau,\vec{J}) - [H_1(\rho_1,\tau_1,\vec{J}_1) + H_2(\rho_2,\tau_2,\vec{J}_2)]\}d\vec{r}, \quad (1)$$

with H as the Skyrme Hamiltonian density. As already stated above, both τ and $\vec{J}$ are further functions of ρ, included here up to second order, and, depending on the approximation used, for the composite system, in:
(a) Sudden approximation,

$$\tau(\rho) = \tau(\rho_1+\rho_2) = \tau(\rho_{1n}+\rho_{2n}) + \tau(\rho_{1p}+\rho_{2p})$$
$$\vec{J}(\rho) = \vec{J}(\rho_1+\rho_2) = \vec{J}(\rho_{1n}+\rho_{2n}) + \vec{J}(\rho_{1p}+\rho_{2p}) \quad (2)$$

and (b) Frozen approximation,

$$\tau = \tau_1(\rho_1) + \tau_2(\rho_2), \quad \vec{J} = \vec{J}_1(\rho_1) + \vec{J}_2(\rho_2), \quad (3)$$

with $\tau_i(\rho_i) = \tau_{in}(\rho_{in}) + \tau_{ip}(\rho_{ip})$, and $\vec{J}_i(\rho_i) = \vec{J}_{in}(\rho_{in}) + \vec{J}_{ip}(\rho_{ip})$.

Introducing slab approximation, we write Eq. (1) as nuclear proximity potential [4]

$$V_N(R) = 2\pi\bar{R}\int_{s_0}^{\infty} e(s)ds = 2\pi\bar{R}\int\{(H(\rho) - [H_1(\rho_1) + H_2(\rho_2)])\}dZ. \quad (4)$$

with $R = R_{01}(\alpha_1, T) + R_{02}(\alpha_2, T) + s$. Here, R_{01} and R_{02} are the temperature (T) dependent radii of two deformed and oriented nuclei, separated by s with a minimum s_0-value.

For nuclear density ρ_i, we use the T-dependent Fermi density distribution

$$\rho_i(Z_i) = \rho_{0i}(T)\left[1 + \exp\frac{Z_i - R_{0i}(T)}{a_i(T)}\right]^{-1}, \quad (5)$$

with $-\infty \le Z \le \infty$, $Z_2 = R - Z_1$ and

$$\rho_{0i}(T) = \frac{3A_i}{4\pi R_{0i}^3(T)}\left[1 + \frac{\pi^2 a_i^2(T)}{R_{0i}^2(T)}\right]^{-1}, \quad (6)$$

with nucleon densities ρ_{iq} further defined as

$$\rho_{in} = (N_i/A_i)\rho_i, \quad \rho_{ip} = (Z_i/A_i)\rho_i \quad (i = 1, 2).$$

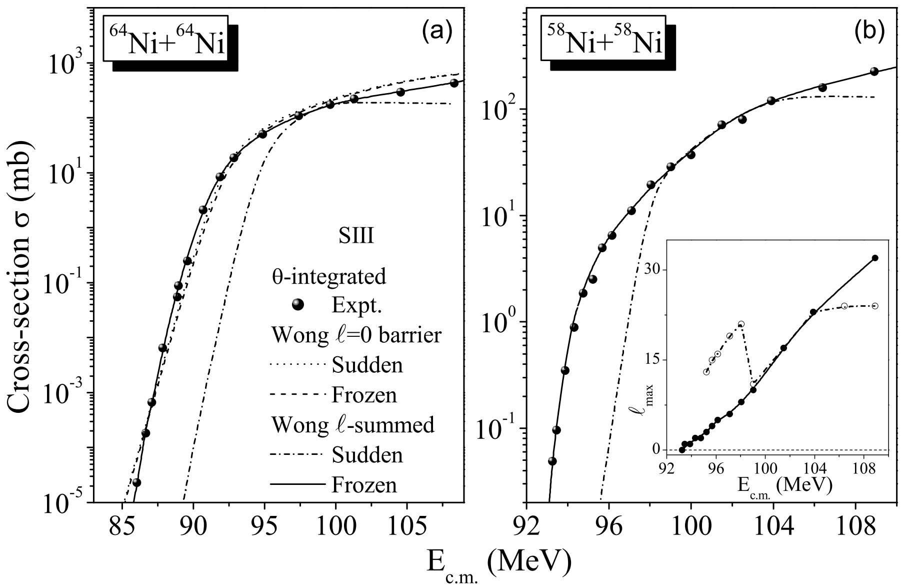

FIG. 1: Cross-sections fitted under sudden- and frozen-densities using ℓ=0 barrier based Wong formula and Wong ℓ-summed expression for SIII Skyrme force. Inset shows the variation of ℓ_{max} with $E_{c.m.}$.

Wong formula

Wong [3] defines the fusion cross-section for two deformed and oriented nuclei lying in same planes, and colliding with center-of-mass (c.m.) energy $E_{c.m.}$, in terms of angular-momentum ℓ partial waves, as

$$\sigma(E_{c.m.},\theta_i) = \frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{max}}(2\ell+1)P_\ell(E_{c.m.},\theta_i), \quad (8)$$

with $k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}}$, and μ as the reduced mass. Here, P_ℓ is the transmission coefficient for each ℓ which describes the penetration of barrier $V_\ell(R, E_{c.m.}, \theta_i)$ calculated in Hill-Wheller approximation. An explicit summation over ℓ in Eq. (8) requires the complete ℓ-dependent potentials $V_\ell(R, E_{c.m.}, \theta_i)$, with ℓ_{max} to be determined empirically.

Wong [3] carried out the ℓ-summation in Eq. (8) approximately under the conditions of using only ℓ=0 quantities, and on replacing the summation by an integral, obtained

$$\sigma = \frac{{R_B^0}^2\hbar\omega_0}{2E_{c.m.}}\ln\left[1+\exp\left(\frac{2\pi}{\hbar\omega_0}(E_{c.m.}-V_B^0)\right)\right] \quad (9)$$

which on integrating over θ_i gives $\sigma(E_{c.m.})$.

Calculations and results

Fig. 1, for the case of ℓ-summed Wong expression, show a point to point fit to data for the frozen-density approximation, with $\ell_{max}(E_{c.m.})$ varying smoothly, as illustrated in the inset of Fig. 1(b). On the other hand, the same could not be achieved for the sudden-density, still requiring a modification of the barrier at both the below- and above-barrier energies. This means that the frozen-density gives the appropriate barriers for the phenomenon of hindrance, observed for these reactions in coupled channel calculations [5], to be explained simply on the basis of ℓ-summed Wong expression.

References

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