# Geometric characteristics of the nuclei

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## Introduction

The densities and its parameters play an important role in describing the geometric characteristics of the nuclei. For medium and heavy nuclei, the radius parameter of the Woods-Saxon (W-S) form of density is a fundamental quantity of geometric importance. Hence, it has been expressed as a function of mass number A in many ways. Here, we compare the different forms of this radius parameter of the W-S form of density and the r. m. s. radii of a large number of medium and heavy nuclei with their respective experimental values.

In our earlier work [1], the radius 'R' and diffuseness 'a' of a W-S form of charge density are chosen to be of the form [2]:

$R = C_0 + C_1 A^{1/3} + C_2 (N - Z) A^{-1}$ (1)

and $a = A_0 + A_1 (N - Z) A^{-1}$, (2)

where N and Z are the number of neutrons and protons respectively. The different sets of the coefficients $C_0$, $C_1$, $C_2$, $A_0$ and $A_1$ are obtained [1] from the fitting of the experimental charge density [3] over a wide range of mass numbers. The coefficients which give the least $\chi^2$ for Λ-binding energies of medium and heavy hypernuclei are chosen as the best set [1]. With these coefficients the half-way radius 'R' of the charge density for a large number of nuclei, with mass number varying from 20 to 243, is calculated and compared with the experimental values. We also compare it with our calculated nucleon density half-way radius [4].

In ref. [4], the point nucleon density $\rho_N(r)$, assumed to be of W-S form, is expressed as

$\rho_N(r) = (Z/A)\, \rho_P(r) + (N/A)\, \rho_n(r)$ , (3)

where both the point proton density $\rho_p(r)$ and point neutron density $\rho_n(r)$ are taken to be of W-S form, with their respective radius and diffuseness parameterization taken from ref.[2]. The coefficients of 'R' and 'a' of $\rho_N(r)$ are obtained [4] by least square fitting of eq. (3), over a large mass number range of medium and heavy nuclei.

The half-way radius obtained by Millener et al. [5], which gives a fairly good account of the Λ-binding energies of the ground as well as the excited states of medium and heavy hypernuclei, is given as:

$R = (1.128 + 0.439\, A^{-2/3})\, A^{1/3}$. (4)

Another half-way radius

$R = (1.144 - 1.276\, A^{-2/3})\, A^{1/3}$, (5)

obtained from the fit to experimental charge radii, is given in ref. [5]. Our calculated 'R' values [1, 4] are also compared with these half-way radii (eq. (4) and (5)), for all the medium and heavy nuclei considered here.

The r. m. s. radius for a Fermi distribution can be written as [5]

$< r^2 >^{1/2} = \{(3/5)\, [R^2 + (7\pi^2 a^2)/3]\}^{1/2}$. (6)

From the 'R' and 'a' of charge density [1], we calculate the r. m. s radii $< r^2 >^{1/2}$ of a large range of medium and heavy nuclei. These calculated $< r^2 >^{1/2}$ are in good agreement with the experimental values [3, 6] as well as with those calculated by Usmani et al [7]. They have studied the changes in the r. m. s. radii which a Λ hyperon induces in nuclei.

## Result and Discussion

The calculated charge density half-way radius is plotted as 1 in Fig. 1, and is found to agree fairly well with the experimental values. The calculated nucleon density half-way radius is also plotted in Fig. 1 as 2 and, as expected, is found to be slightly more than the half-way radius of the charge density for all the nuclei considered here, with mass number varying from 20 to 243. The half-way radius (eq. 4) given by Millener et al. [5], is plotted as 3, in Fig.1. Another half-way radius (eq. (5)) as given in ref.

[5], is plotted as 4, in Fig.1. The experimental 'R' values [3] are also plotted in the Fig.1 for comparison. The 'R' values calculated by us, as well as by Millener et al. [5] are found to reproduce the data quite well. However, our parameterization of 'R' (eq. (1)), which contains an asymmetry term to distinguish between neutron and proton distribution, is expected to give better results, particularly for heavy nuclei.

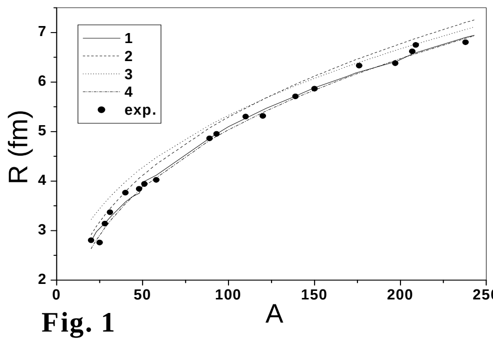


**Fig. 1**

The charge density obtained with 'R' and 'a' from [1] is plotted in Fig. 2(a) for six nuclei (A= 28, 40, 51, 89,150,208), to see the difference in the 'R' with increasing mass numbers. The difference in the 'R' for $^{28}$Si and $^{208}$Pb is found to be ~2.5 fm. Although the major contribution to the value of 'R' comes from the $A^{1/3}$ term, however, the asymmetry term becomes significant for heavy nuclei.

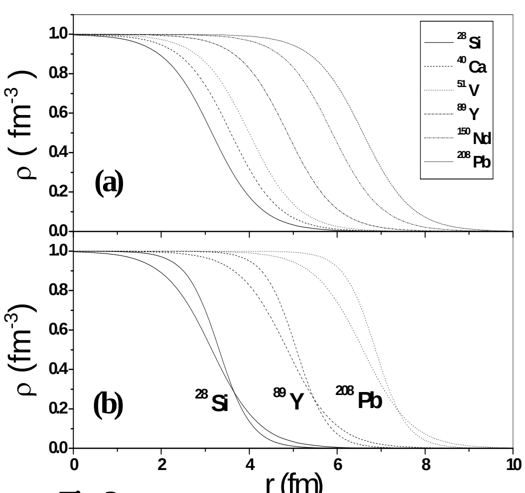


**Fig. 2**

We also observe that 'R' for nucleon density is larger while 'a' is smaller compared to those of the charge density of the same nuclei, as shown in Fig. 2(b), for $^{28}$Si, $^{89}$Y and $^{208}$Pb. This may be attributed to the 'saturation' properties of the nucleus.

Our calculated $< r^2 >^{1/2}$, represented by solid line (shown as 1), in Fig. 3, are found to be quite close to the available experimental values [3, 6] for the medium and heavy nuclei. The slight deviation for heavy nuclei may be attributed to the inadequate experimental data in this mass number range. Our results also agree well with those obtained by Usmani et al. [7] for compression modulus K = 250 (represented as 2, in Fig. 3). This indicates that the core polarization for medium and heavy nuclei is quite small as is shown by them [7].

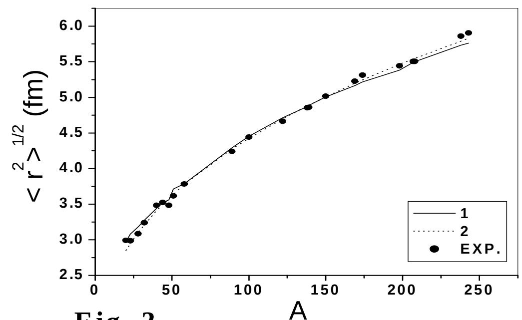


**Fig. 3**

Thus, we conclude that our calculated 'R' and r. m. s. radii $< r^2 >^{1/2}$ give a fairly good reproduction of the respective experimental data, over a large range of medium and heavy nuclei and hence represent the geometry of nuclear densities fairly well, on which the various potentials and energy calculations depend. However, more experimental data on heavy nuclei is highly desirable.

## References


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