Tunneling through a composite potential and understanding deep sub-barrier fusion reactions

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In the field of heavy-ion reactions, one of the major issues that has attracted attention in recent time is the steep falloff of the measured fusion cross sections (σ_f) which has been observed in many heavy-ion systems at energy far below the Coulomb barrier. The hindrance is often so strong that the results of σ_f presented in the form of L-factor $L(E) = \frac{d\ ln(E\sigma_f)}{dE}$ shows a steep rise and the S-factor $S(E) = \sigma_f\ E\ exp(2\pi\eta)$ develops a bending at low energies. Here, E stands for center-of-mass energy and η indicates the Sommerfeld parameter. In this contribution we address this problem of deep sub-barrier fusion with a formulation based on a one-dimensional barrier transmission across a composite potential barrier which closely resembles the outer barrier generated by the resultant nucleus-nucleus potential. We construct such a barrier potential first of its kind by placing side by side two potentials of same form $v_j(x)$, j=1, 2. The full potential is expressed as $V(x) = V_{01}v_1(x)\theta(x) + V_{02}v_2(x)\theta(-x)$, where the two step functions have the property $\theta(x \le 0) = 0$ and $\theta(x > 0) = 1$. Further, $v_1(x) = 1 - \left[\frac{1-exp(x/a_1)}{1-c_1 exp(x/a_1)}\right]^2$, $c_1 > 1$, $v_2(x) = 1 - \left[\frac{1-exp(-x/a_2)}{1-c_2 exp(-x/a_2)}\right]^2$, $c_2 > 1$. The curvature of the barrier on either side is given by $C_{01} = 2V_{01}/[a_1^2(1-c_1)^2]$ or $C_{02} = 2V_{02}/[a_2^2(1-c_2)^2]$. To have a single barrier of height V_B we take $V_{01} = V_{02} = V_B$. The s-wave (ℓ=0) full potential barrier along with other two barriers for partial waves ℓ=12, 20 for the ^{48}Ca+^{96}Zr system is shown in figure 1. We obtain exact solution of the respective potentials in the region $x > 0$ and $x < 0$ on either side of the barrier. By using proper boundary conditions on these solutions we express the transmission coefficient $T(E) =\mid \tau(E) \mid^2$ across the full barrier with the transmission amplitude given by

$$\tau(E) = H(\frac{D_r - D_i}{D_r - D_t})(\frac{\kappa'}{\kappa}). \quad (1)$$

$$D_i = (\frac{c_1}{a_1})\Big[\frac{ir}{c_1} + \frac{ir + is}{1 - c_1} + \frac{1}{(c_1 - 1)c_1}\frac{A_i B_i}{C_i}F_i\Big], \quad (2)$$

where $F_i = \frac{F(A_i+1, C_i-B_i, C_i+1, 1/c_1)}{F(A_i, C_i-B_i, C_i, 1/c_1)}$, $A_i = \frac{1}{2} + ir - it + is$, $B_i = \frac{1}{2} + ir + it + is$, $C_i = 1 + 2is$, $s = [r^2 + q^2(b^2-1)]^{1/2}$, $t = [q^2(b+1)^2 - \frac{1}{4}]^{1/2}$, $r = ka_1$, $b = -1/c_1$, $q^2 = V_{01}/\Delta$, $\Delta = \frac{\hbar^2}{2ma_1^2}$, $k = \sqrt{2mE/\hbar^2}$, $\kappa = \sqrt{\frac{2m}{\hbar^2}[E + (b^2-1)V_{01}]}$.

$$D_r = (\frac{c_1}{a_1})\Big[-\frac{ir}{c_1} - \frac{ir + is}{1 - c_1} + \frac{1}{(c_1 - 1)c_1}\frac{A_r B_r}{C_r}F_r\Big], \quad (3)$$

where $F_r = \frac{F(A_r+1, C_r-B_r, C_r+1, 1/c_1)}{F(A_r, C_r-B_r, C_r, 1/c_1)}$, $A_r = \frac{1}{2} - ir - it - is$, $B_r = \frac{1}{2} - ir + it - is$, $C_i = 1 - 2is$.

$$D_t = (\frac{c_2}{a_2})\Big[-\frac{ir'}{c_2} - \frac{ir' + is'}{1 - c_2} + \frac{1}{(c_2 - 1)c_2}\frac{A_t B_t}{C_t}F_t\Big], \quad (4)$$

where $F_t = \frac{F(A_t+1, C_t-B_t, C_t+1, 1/c_2)}{F(A_t, C_t-B_t, C_t, 1/c_2)}$, $A_t = \frac{1}{2} - ir' - it' - is'$, $B_t = \frac{1}{2} - ir' + it' - is'$, $C_t = 1 - 2is'$, $s = [r'^2 + q'^2(b'^2-1)]^{1/2}$, $t = [q'^2(b'+1)^2 - \frac{1}{4}]^{1/2}$, $r = ka_2$, $b' = -1/c_2$, $q'^2 = V_{01}/\Delta'$, $\Delta' = \frac{\hbar^2}{2ma_2^2}$, $\kappa' = \sqrt{\frac{2m}{\hbar^2}[E + (b'^2-1)V_{01}]}$.

$H = \frac{(1-1/c_1)^{1/2-it}}{(1-1/c_2)^{1/2-it'}}\frac{F(A_i, C_i-B_i, C_i, 1/c_1)}{F(A_t, C_t-B_t, C_t, 1/c_2)}$.

In order to consider the various centrifugal barrier generated by different partial waves ℓ, we need to replace the height V_B by the effective barrier height $V_B^\ell = V_B + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{R_B^2}$, where

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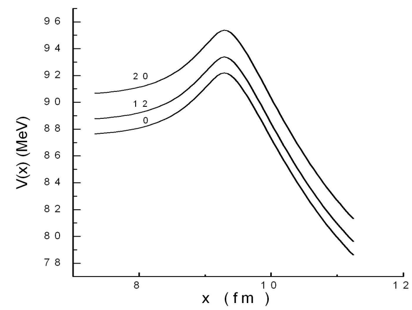

FIG. 1: Plot of full potential barrier for three different partial wave ℓ=0, 12, 20.

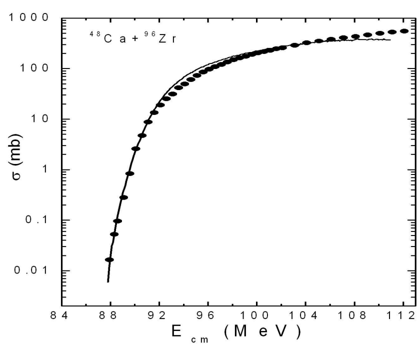

FIG. 2: Plot of fusion cross section as function of center of mass energy.

V_B and R_B indicate the height and radial position of the s-wave barrier, respectively. As a result of this, the expression (1) gives transmission coefficient T_ℓ for different ℓs. The resultant value of σ_f at a given energy is obtained by using $\sigma_f = \frac{\pi}{k^2}\Sigma_{\ell=0}^{\infty}(2\ell+1)T_\ell$. For the system $^{48}Ca + ^{96}Zr$, the potential parameters are taken as V_B=92.2 MeV, R_B=9.29 fm, c_1 =2, a_1=2 fm, c_2 =4.42 and a_2=0.52 fm. The calculated results (solid curves) of σ_f, L-factor and S-factor are compared with the corresponding experimental results (solid dots) taken from [1] in figures 2, 3(a) and 3(b), respectively. In general, the explanation of the data in each situation is found to be quite good. In particular, the features of steep rise in L-factor and result of bending in S-factor observed in the deep sub-barrier region of energy are explained with remarkable success.

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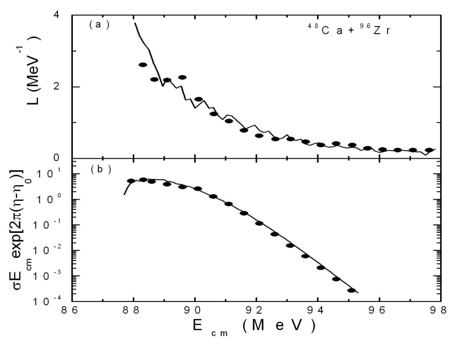

FIG. 3: Energy variation of L-factor=$\frac{dln(\sigma_f \ E_{cm})}{dE_{cm}}$ and S-factor=$\sigma_f E_{cm}\ exp[2\pi(\eta-\eta_0)]$ with η_0 = 75.8.

[1] H. Esbensen and C. L. Jiang, *Phys. Rev.* C **79** 064619 (2009).