Tunneling through a composite potential and understanding deep sub-barrier fusion reactions

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In the field of heavy-ion reactions, one of the major issues that has attracted attention in recent time is the steep falloff of the measured fusion cross sections ($\sigma_f$) which has been observed in many heavy-ion systems at energy far below the Coulomb barrier. The hindrance is often so strong that the results of $\sigma_f$ presented in the form of L-factor $L(E) = e^{\Delta ln(\sigma_f)}$ shows a steep rise and the S-factor $S(E) = \sigma_f E \exp(2\pi\eta)$ develops a bending at low energies. Here, $E$ stands for center-of-mass energy and $\eta$ indicates the Sommerfeld parameter. In this contribution we address this problem of deep sub-barrier fusion with a formulation based on a one-dimensional barrier transmission across a composite potential barrier which closely resembles the outer barrier generated by the resultant nucleus-nucleus potential. We construct such a barrier potential first of its kind by placing side by side two potentials of same form $v_j(x)$, $j=1, 2$. The full potential is expressed as $V(x) = V_1 v_1(x) \theta(x) + V_2 v_2(x) \theta(-x)$, where the two-step functions have the property $\theta(x) = 0$ and $\theta(x) = 1$. Further, $v_1(x) = 1 - \left[1 - \exp(x/a_1)\right]^2$, $c_1 > 1$, $v_2(x) = 1 - \left[1 - \exp(-x/a_2)\right]^2$, $c_2 > 1$. The curvature of the barrier on either side is given by $C_01 = 2V_01/[a_1^2(1-c_1)^2]$ or $C_02 = 2V_02/[a_2^2(1-c_2)^2]$. To have a single barrier of height $V_B$ we take $V_01 = V_02 = V_B$. The s-wave ($l=0$) full potential barrier along with other two barriers for partial waves $l=12, 20$ for the $^{48}$Ca+$^{96}$Zr system is shown in figure 1. We obtain exact solution of the respective potentials in the region $x > 0$ and $x < 0$ on either side of the barrier. By using proper boundary conditions on these solutions we express the transmission coefficient $T(E) = \tau(E)$ across the full barrier with the transmission amplitude given by

$$\tau(E) = H \left(\frac{D_r - D_i}{D_r - D_t}\right) \frac{\kappa'}{\kappa}, \quad (1)$$

$$D_i = \left(\frac{c_1}{a_1}\right) \left[\frac{ir + is}{c_1 - 1} + \frac{1}{(c_1 - 1)c_1} A_i B_i F_i\right], \quad (2)$$

where $F_i = \frac{F(A_i + 1, C_i - B_i, C_i + 1, 1/c_1)}{F(A_i, C_i - B_i, C_i, 1/c_1)}$, $A_i = \frac{1}{2} + i r - i t + is, B_i = \frac{1}{2} + i r + it + is, C_i = 1 + 2is, s = [r^2 + q^2(b^2 - 1)]^{1/2}, t = [q^2(b + 1)^2 - \frac{1}{4}]^{1/2}, r = ka_1, b = -1/c_1, q^2 = V_01/\Delta, \Delta = \frac{h^2}{2ma_1^2}, k = \sqrt{2mE/\kappa}, \kappa = \sqrt{\frac{2m}{h^2}[E + (b^2 - 1)V_01]}.$

$$D_r = \left(\frac{c_1}{a_1}\right) \left[-\frac{ir}{c_1} - \frac{ir + is}{1 - c_1} + \frac{1}{(c_1 - 1)c_1} A_i B_r F_r\right], \quad (3)$$

where $F_r = \frac{F(A_i + 1, C_i - B_i, C_i + 1, 1/c_1)}{F(A_i, C_i - B_i, C_i, 1/c_1)}$, $A_r = -\frac{1}{2} - i r - i t - is, B_r = \frac{1}{2} - i r + it - is, C_i = 1 + 2is$.

$$D_t = \left(\frac{c_2}{a_2}\right) \left[-\frac{ir'}{c_2} - \frac{ir' + is'}{1 - c_2} + \frac{1}{(c_2 - 1)c_2} A_i B_t F_t\right], \quad (4)$$

where $F_t = \frac{F(A_i + 1, C_i - B_i, C_i + 1, 1/c_2)}{F(A_i, C_i - B_i, C_i, 1/c_2)}$, $A_i = \frac{1}{2} - i r' - i t' - is', B_i = \frac{1}{2} - i r' + it' - is', C_i = 1 - 2is', s = [r'^2 + q^2(b^2 - 1)]^{1/2}, t = [q^2(b' + 1)^2 - \frac{1}{4}]^{1/2}, r = ka_2, b' = -1/c_2, q^2 = V_01/\Delta', \Delta' = \frac{h^2}{2ma_2^2}, \kappa' = \sqrt{\frac{2m}{h^2}[E + (b^2 - 1)V_01]}.$

In order to consider the various centrifugal barrier generated by different partial waves $l$, we need to replace the height $V_B$ by the effective barrier height $V_B^e = V_B + \frac{2\hbar^2}{mR^2} l(l+1)$, where

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FIG. 1: Plot of full potential barrier for three different partial wave $\ell=0, 12, 20$.

FIG. 2: Plot of fusion cross section as function of center of mass energy.

$V_B$ and $R_B$ indicate the height and radial position of the s-wave barrier, respectively. As a result of this, the expression (1) gives transmission coefficient $T_\ell$ for different $\ell$s. The resultant value of $\sigma_f$ at a given energy is obtained by using $\sigma_f = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_\ell$. For the system $^{48}$Ca + $^{96}$Zr, the potential parameters are taken as $V_B=92.2$ MeV, $R_B=9.29$ fm, $c_1=2$, $a_1=2$ fm, $c_2=4.42$ and $a_2=0.52$ fm. The calculated results (solid curves) of $\sigma_f$, L-factor and S-factor are compared with the corresponding experimental results (solid dots) taken from [1] in figures 2, 3(a) and 3(b), respectively. In general, the explanation of the data in each situation is found to be quite good. In particular, the features of steep rise in L-factor and result of bending in S-factor observed in the deep sub-barrier region of energy are explained with remarkable success.

$$\sigma_f = \sigma_f(E_{cm}) = \sigma_f(E_{cm}) \exp[2\pi(\eta - \eta_0)]$$ with $\eta_0 = 75.8$.

FIG. 3: Energy variation of L-factor and S-factor observed in the deep sub-barrier region of energy are explained with remarkable success.