Fission as diffusion of a Brownian particle with variable inertia

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A hot compound nucleus (CN) undergoes fission when sufficient kinetic energy accumulates on a collective fission degree of freedom. A suitable model to describe such a process is the motion of a Brownian particle in a heat bath. The dynamics of such a system is governed by the appropriate Langevin equations or equivalently by the corresponding Fokker-Planck equation. The collective kinetic energy of the CN in these equations contain an inertia term which also depend on the collective coordinates [1]. The CN inertia thus becomes shape-dependent.

Kramers solved the Fokker-Planck equation analytically and obtained the stationary fission width where the collective inertia was assumed to be shape-independent and constant[2]. It is recently shown that Kramers' width also gives the correct fission width for cases where the shape-dependence is not too strong[3]. In the present work, we would explore the extent to which the Kramers' width retains its validity when the shape-dependence of the inertia is made stronger. To this end, we shall compare Kramers' fission width with that obtained from the numerical solution of Langevin equations where shape-dependent inertia is employed. The width from the Langevin equations is considered to represent the true fission width.

We solve the Langevin equations in one dimension considering the elongation parameter c as the relevant collective coordinate and using the collective potential V and different empirical forms of shape-dependence of inertia m as shown in Fig.1 for ^{224}Th.

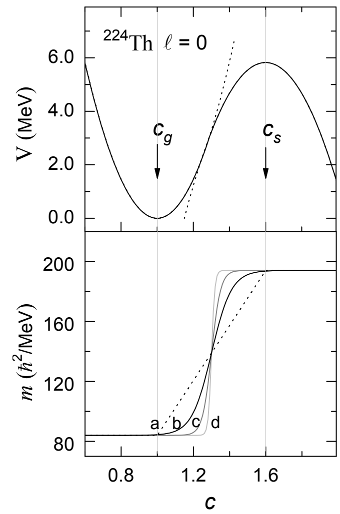

FIG. 1: Collective potential for ^{224}Th (upper panel) constructed using two parabolas and different shapes of inertia (lower panel).

The time-dependent fission rates from Langevin equations calculated with each shape-dependent inertia are shown in Fig.2 and they are found to be very similar to each other. All the stationary values of the fission widths are in close agreement with the Kramers' width[2]

$$\Gamma_K = \frac{\hbar\omega_g}{2\pi} e^{-V_B/T} \left\{ \sqrt{1 + \left(\frac{\eta}{2\omega_s}\right)^2} - \frac{\eta}{2\omega_s} \right\} \quad (1)$$

where ω_g and ω_s define the frequencies of the harmonic oscillator potentials at c_g and c_s respectively, V_B is the height of the fission barrier and η is the dissipation strength.

The Kramers' width represents the diffusion

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rate of the Brownian particles across the fission barrier governed by the appropriate Liouville equation.

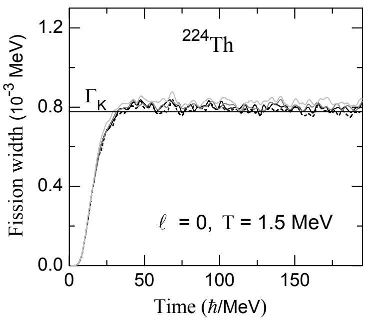

FIG. 2: Time-dependent fission widths from Langevin equations for different shapes of m as given in Fig. 1.

In deriving Eq.(1), it is assumed that the inertia derivative term in the Liouville equation is small. We however find in Fig.2 that Kramers' width holds good even for the case where the inertia derivative is very large (case d in Fig.1).

We subsequently perform Langevin calculations with a number of shape-dependent inertias as shown in Fig.3 (upper panel). All of them represent a sharp but continuous rise of the inertia value but at different transition points (c_t). The variation of the stationary values of the fission rate from Langevin calculation (Γ_L) with c_t is also given in Fig.3 (lower panel). The Kramers' widths obtained with inertia values fixed at either the ground state deformation or at the saddle are also shown in this figure (Γ_K^g and Γ_K^s, respectively). It is observed that for $c_t < c_g$, Γ_L approaches Γ_K^s while for $c_t > c_s$, Γ_L is close to Γ_K^g. This trend is expected since most part of the Langevin dynamics takes place with inertia at saddle for $c_t < c_g$ while the Browninan particles mostly move with ground state inertia for $c_t > c_s$. We also observe that $\Gamma_L \sim \Gamma_K^{gs}$ calculated for c_t values midway between c_g and c_s and this can be explained as follows. The dynamics near the saddle can be well described by the Liouville

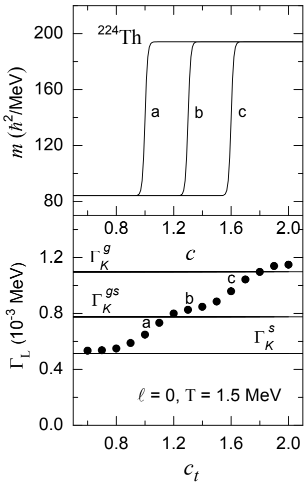

FIG. 3: Shape-dependent inertia (upper panel) and the corresponding Γ_L (lower panel) for different values of c_t.

equation with a constant inertia m_s and the inertia derivative term as zero when $c_t << c_s$. Similarly, the Brownian particle moves with a constant inertia m_g when $c_t >> c_g$. Both these conditions are approximately met when c_t is in the midway between c_g and c_s. In the limit of large friction, which we are considering here, the local equilibration is fast and the large inertia dependence over a narrow region which is far from both c_g and c_s does not affect the dynamics either near the ground state or at saddle deformations.

We therefore conclude that the Kramers' fission width remains valid even for very strong shape dependence of inertia provided the variation takes place at shapes midway between the ground state and saddle deformations.

References

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