Elastic scattering of 1.0 GeV protons on \(^{40,42,44,48}\)Ca

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Introduction

One of the main objectives in the analyses of hadron-nucleus collisions at intermediate and high energies has been to extract information about the neutron density distribution, as the proton density distribution is fairly reliably known from the electron scattering experiments. On theoretical front, the Glauber model is found to provide a quite successful tool for such studies. One of the attractive features of Glauber model is that it connects the directly measurable (free) hadron-nucleon scattering amplitude to the hadron-nucleus one in a mathematically tractable way. In a recent work[1], our analysis of 1 GeV proton elastic scattering from light (N=Z) nuclei has demonstrated that the consideration of the better choice of the NN amplitude is able to provide a satisfactory account of the experimental data up to the available range of momentum transfer.

In this work, we present the (Coulomb modified) Glauber model analysis of the elastic scattering of 1 GeV protons on Ca isotopes using the similar form of the NN amplitude as proposed in ref.[1]. Our aim is to see how far the similar consideration[1] account for the data on the Ca isotopes, and what could be said about the neutron density distribution from such studies.

Formulation

Following Ahmad and Auger[2], the Coulomb modified[1] correlation expansion of the Glauber amplitude for describing the elastic scattering of protons with momentum \(k\) from a target nucleus in the ground state \(|\psi_0\rangle\) takes the form:

\[
F_0(q) = \frac{i k}{2\pi} \int d^2 b e^{i q \cdot b} e^{i (x_\mu (\vec{b}) + x_\nu (\vec{b}))} \\
[1 - (1 - \Gamma_0)^4 e^{i \chi (\vec{b})}],
\]

\[
F_1(q) = -\frac{i k}{2\pi} \int d^2 b e^{i q \cdot b} e^{i (x_\mu (\vec{b}) + x_\nu (\vec{b}))} \\
\times \langle \psi_0 | (1 - \Gamma_0)^{k-1} \sum_i \sum_j \gamma_i \gamma_j \cdots \gamma_k | \psi_0 \rangle
\]

with \(\gamma_j = \Gamma_0 (\vec{b}) - \Gamma_{NN} (\vec{b} - \vec{s}_j)\),

\[
\Gamma_0 (\vec{b}) = \langle \psi_0 | \Gamma_{NN} (\vec{b} - \vec{s}_j) | \psi_0 \rangle
\]

where \(s_j\) is the projection of \(j\)th target nucleon coordinate \(r_j\) onto a plane perpendicular to \(k\), and the NN profile function \(\Gamma_{NN}\) is related to the NN amplitude \(f_{NN}\) as:

\[
\Gamma_{NN} (B) = \frac{1}{2\pi k} \int d^2 q e^{-\frac{1}{2} q^2} f_{NN} (\vec{q}).
\]

The quantities \(F(1)\), \(F_{20}\), and \(F_{21}\) are the same as defined in ref.[3], and the distance of closest approach \(b^*\), that takes into account the deviation in the trajectory because of the Coulomb field, has the same expression as used in ref.[4].

In the present work, we restrict ourselves up to \(F_2\) in the expression for \(F_{00}\) as it provides a leading correction to the optical limit[5] term \(F_0\). More explicitly

\[
F_2(q) = \frac{i}{8\pi^3 k} \frac{A (A - 1)}{2!} \int d^2 b e^{i q \cdot b} e^{i (x_\mu (\vec{b}) + x_\nu (\vec{b}))} \\
(1 - \Gamma_0)^{d-2} [G_2 - 2 G_1^3],
\]

\[
G_2 = \int d^2 q_1 d^2 q_2 e^{i (q_1 - q_2) \cdot \vec{b}} f_{NN}(q_1) f_{NN}(q_2) F_{22}(q_1, q_2)
\]

\[
G_1 = \int d^2 q e^{-\frac{1}{2} q^2} f_{NN}(q) F(q)
\]

The quantities \(F(q)\) and \(F_{22}(q_1, q_2)\) in the above expressions are the one- and two-body (intrinsic) form factors respectively.

For the intrinsic two-body form factor \(F_{22}(q_1, q_2)\), we use the following expression as derived in ref.[5],

\[
F_{22}(q_1, q_2) = K (q_1 + q_2) \\
\left[ \frac{F(q_1) F(q_2)}{K(q_1) K(q_2)} \right]^{\frac{1}{2}} D_{2M}(q_1 + q_2),
\]

where \(K(q)\) is the c.m. correlation correction.
factor, and $\tilde{g}$, and $D_m(q)$ are the same as defined in ref.[5].

**Results and Discussion**

Following the above mentioned approach, we analyse the elastic angular distribution of 1 GeV protons on Ca isotopes. The inputs needed in the calculation are the elementary NN amplitude, the nuclear form factors, and the oscillator constants[5]. Following ref.[6], we parametrize the NN amplitude as follows:

$$f_{NN}(q) = \frac{i k \sigma}{4 \pi} \sum_{n=0}^{\infty} A_{n+1} \left( \frac{\sigma}{4 \pi f^2} \right)^{n+1} \frac{(1-i \rho)^{n+1}}{n+1} e^{-q^2/(n+1)} +$$

$$\left( \frac{q}{4 m} \right)^{1/2} \frac{\sigma}{4 \pi f^2} \frac{[D d(-i \omega)]^{n+1}}{n+1} e^{q^2/(2 a_0)}$$

(11)

where

$$A_{n+1} = \frac{A_1}{n(n+1)} + \frac{A_1}{(n-1)n} + \frac{A_1}{(n-2)(n-1)} + \ldots + \frac{A_1}{1.2}$$

with $A_1=1$. This amplitude has 6 adjustable parameters $\sigma$, $\rho$, $\beta^2$, $D_s$, $\rho_s$ and $\beta_s^2$, whose values are chosen under the conditions that at 1GeV incident energy (i) the optical theorem be valid, (ii) the ratio $\text{Re} f_{NN}(0)/\text{Im} f_{NN}(0)$ be equal to the experimental value, and (iii) the experimental elastic angular distribution and polarization data for NN scattering be correctly reproduced.

For computational simplicity we parametrize the required nuclear form factors as a sum of Gaussians:

$$F(q) = \sum_i a_i e^{-b_i q^2}$$

where $a_i$ and $b_i$ are parameters whose values are obtained by fitting the electron scattering form factors after correcting for the finite size of the proton. The values of oscillator constants are taken from ref.[7].

The results of calculations are presented in Fig.1. These calculations involve similar density distribution for protons and neutrons. It is seen that, except beyond the second diffraction minima, the data are nicely reproduced in all the cases. As pointed out in ref.[1], the results on $^{40}\text{Ca}$ supports different density distributions for protons and neutrons beyond the second diffraction minimum. One hopes that the similar considerations may improve the situation in the case of Ca isotopes. The quantitative calculations involving realistic distributions for protons and neutrons in Ca isotopes are in progress.

**References**