**Effective ΛN and ΛΛ Interactions with the Skyrme-Hartree-Fock Theory**

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The upcoming experimental facilities at KEK, JLAB, J-PARC, and other sites will soon provide precise measurements on the many-body hadronic systems of Hypernuclear Physics utilizing the strangeness degrees of freedom. The investigation of Hypernuclear physics mainly focused on the single and double Λ hypernuclei at this stage [1]. Recently, this subject has generated theoretical interest and, apart from the exploration of hypernuclear matter at supranuclear densities, several attempts have been made to study the structure properties of hypernuclei in many-body mean field theories and lattice QCD calculations.

In this paper, we have used an extended Skyrme-Hartree-Fock theory to calculate the interaction strength parameters of ΛN effective interaction, ΛNN three-body interaction, ΛΛ effective interaction, and ΛΛN three-body interaction. The parameters sets are obtained so as to reproduce the Λ single particle energies of single Λ hypernuclei and, the binding energies of double Λ hypernuclei spanning a wide range in the periodic table. The total energy density functional (EDF) of Λ hypernucleus with in the extended density dependent Skyrme Hartree-Fock theory is expressed as,

\[ \mathcal{E}_\Lambda = \mathcal{E}_{NN}(\rho_n, \rho_p, \tau_n, \tau_p, J_n, J_p) + \mathcal{E}_{\Lambda N}(\rho_n, \rho_p, \rho_{\Lambda}, \tau_{\Lambda}) + \mathcal{E}_{\text{pair}}(v_p, v_n) + \mathcal{E}_{\Lambda N}^R(\rho_n, \rho_p, \rho_{\Lambda}) \]  

Where, \( \mathcal{E}_{NN} \) is the original Skyrme Hartree-Fock nuclear Hamiltonian density based upon the nucleon-nucleon interactions. We employ SLy4 [2] Skyrme parameterization to calculate the energy density functional of core nucleus \[ \frac{A-1}{Z} \] of hypernucleus. The \( \rho_i \) (i = n, p, and \( \Lambda \)) one body density, \( \tau_i \) kinetic density, \( J_{n/p} \), the spin orbit current operator, and \( v_{n/p} \) are the occupation probabilities of nucleons in the core nucleus.

The contribution to the total energy density functional of hypernucleus due to the presence of Λ hyperon may be written as,

\[ \mathcal{E}_\Lambda = \int d^3rH_{\Lambda N}(r), \]

\[ \mathcal{E}_{\text{pair}}(v_p, v_n) \] is the pairing density functional, and \( \mathcal{E}_{\Lambda N}^R(\rho_n, \rho_p, \rho_{\Lambda}) \) is contribution from rearrangement energy functional.

The total energy density functional for ΛN hypernuclei \[ \Lambda^{+}Z \] is written as,

\[ \mathcal{E}_{\Lambda N}^R = \mathcal{E}_{\Lambda N}^H + \mathcal{E}_\Lambda + \mathcal{E}_{\Lambda N}^R(\rho_n, \rho_p, \rho_{\Lambda}) \]  

The double Λ energy density may be written as,

\[ \mathcal{E}_{\Lambda \Lambda} = \int d^3rH_{\Lambda \Lambda}(r), \]

the Hamiltonian density \( H_{\Lambda \Lambda} \) is written by using two-body and three-body Skyrme interaction forces.

In Table (I), we present the different sets of parameterizations obtained by the methods of successive approximation for χ² minimization. In case of AN effective interaction parameters, our calculated data set is fit to experimental data set consists of Λ single particle energies of hypernuclei, \( \Lambda^{3} \)He, \( \Lambda^{7} \)Li, \( \Lambda^{10} \)Be, \( \Lambda^{11} \)B, \( \Lambda^{12} \)C, \( \Lambda^{13} \)C, \( \Lambda^{16} \)O, \( \Lambda^{17} \)O, \( \Lambda^{28} \)Si, \( \Lambda^{30} \)Si, \( \Lambda^{40} \)Ca, \( \Lambda^{41} \)Ca, \( \Lambda^{51} \)V, \( \Lambda^{56} \)Fe, \( \Lambda^{59} \)Y, \( \Lambda^{88} \)La, and \( \Lambda^{208} \)Pb. We get values of AN potential, \( \mathcal{V}_{AN} \)
TABLE I: A new parameterizations for Λ-N Skyrme potential derived self-consistently by fitting a large set of experimental data of Λ single particle energies for two different values of β.

<table>
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<th>SET</th>
<th>β</th>
<th>u₀ (MeV fm³)</th>
<th>u₁ (MeV fm³)</th>
<th>u₂ (MeV fm⁵)</th>
<th>u₃ (MeV fm³⁺³)</th>
<th>y₀</th>
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</table>

FIG. 1: The difference in the experimental and theoretical calculated single particle energies, ΔεΛ plotted as a function of baryon numbers, A in hypernuclei β = 1/3.

FIG. 2: Same as Fig. (1), for β = 1.

ΔεΛ = εΛ excpt. - εΛ theor. single particle energies in hypernuclei for 1s, 1p, and 1d orbitals for β = 1/3 and 1.0, respectively as a function of total baryon numbers. The relative errors in εΛ is always less than 0.1 for all the hypernuclei data employed in fit. The experimental data for binding energies BΛΛ and ΛΛ interaction energy ∆BΛΛ from KEK E176 Collaboration [3] on light ΛΛ hypernuclei have been employed to search the parameterizations of effective ΛΛ interactions. In this case we get double Λ potential, VΛΛ = 5.00±0.5 MeV at ρ = 0.16fm⁻¹.

In this work we have generated Skyrme interaction strength parameters for single Λ nucleons interaction, and double Λ nucleons effective interactions within Hartree Fock theory by using the method of successive approximations algorithm of χ² minimization. For the first time large set of data have been used in the fit. The parameterizations can be further employed to study the structure properties of all the available hypernuclei, to investigate bulk nuclear matter made up with hyperons at supranuclear density, to construct equation of state at zero and finite temperature of compact objects of astrophysical interest.

References