Effective ΛN and ΛΛ Interactions with the Skyrme-Hartree-Fock Theory

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The upcoming experimental facilities at KEK, JLAB, J-PARC, and other sites will soon provide precise measurements on the many-body hadronic systems of Hypernuclear Physics utilizing the strangeness degrees of freedom. The investigation of Hypernuclear physics mainly focused on the single and double Λ hypernuclei at this stage [1]. Recently, this subject has generated theoretical interest and, apart from the exploration of hypernuclear matter at supranuclear densities, serval attempts have been made to study the structure properties of hypernuclei in many body mean field theories and lattice QCD calculations.

In this paper we have used an extended Skyrme-Hartree-Fock theory to calculate the interaction strength parameters of ΛN effective interaction, ΛNN three body interaction, ΛΛ effective interaction, and ΛΛN three-body interaction. The parameters sets are obtained so as to reproduce the Λ single particle energies of single Λ hypernuclei and, the binding energies of double Λ hypernuclei spanning a wide range in the periodic table. The total energy density functional (EDF) of Λ hypernucleus with in the extended density dependent Skyrme Hartree Fock theory is expressed as,

$$\begin{aligned}\mathcal{E}^H_{1\Lambda} &= \mathcal{E}_{NN}(\rho_n,\rho_p,\tau_n,\tau_p,J_n,J_p)\\ &\quad+\mathcal{E}_{\Lambda N}(\rho_n,\rho_p,\rho_\Lambda,\tau_\Lambda)\\ &\quad+\mathcal{E}_{Pair}(v_p,v_n)+\mathcal{E}^\Lambda_R(\rho_n,\rho_p,\rho_\Lambda).\end{aligned} \quad (1)$$

Where, $\mathcal{E}_{NN}$ is the original Skyrme Hartree-Fock nuclear Hamiltonian density based upon the nucleon-nucleon interactions. We employ SLy4 [2] Skyrme parameterization to calculation the energy density functional of core nucleus ${}^{A-1}_{\Lambda}Z$ of hypernucleus. The ρ_i, (i = n,p, and Λ) one body density, τ_i kinetic density, $J_{n/p}$ the spin orbit current operator, and $v_{n/p}$ are the occupation probabilities of nucleons in the core nucleus. The contribution to the total energy density functional of hypernucleus due to the presence of Λ hyperon may be written,

$$\mathcal{E}_{\Lambda N} = \int d^3r H_{\Lambda N}(r), \quad (2)$$

$\mathcal{E}_{Pair}(v_p, v_n)$ is the pairing density functional, and $\mathcal{E}^\Lambda_R(\rho_n,\rho_p,\rho_\Lambda)$ is contribution from rearrangement energy functional. The total energy density functional for ΛΛ hypernuclei ${}^{A+1}_{\Lambda\Lambda}Z$ is written as,

$$\mathcal{E}^H_{2\Lambda} = \mathcal{E}^H_{1\Lambda} + \mathcal{E}_{\Lambda\Lambda} + \mathcal{E}^{\Lambda\Lambda}_R(\rho_N,\rho_\Lambda) \quad (3)$$

The double Λ energy density may written as,

$$\mathcal{E}_{\Lambda\Lambda} = \int d^3r H_{\Lambda\Lambda}(r), \quad (4)$$

the Hamiltonian density $H_{\Lambda\Lambda}$ is written by using two-body and three body Skyrme interaction forces.

In Table (I), we present the different sets of parameterizations obtained by the methods of successive approximation for χ^2 minimization. In case of ΛN effective interaction parameters, our calculated data set is fit to experimental data set consists of Λ single particle energies of hypernuclei, ${}^{8}_{\Lambda}$He,${}^{9}_{\Lambda}$Li, ${}^{10}_{\Lambda}$Be, ${}^{10}_{\Lambda}$B, ${}^{11}_{\Lambda}$B, ${}^{12}_{\Lambda}$B, ${}^{12}_{\Lambda}$C, ${}^{13}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, ${}^{17}_{\Lambda}$O, ${}^{28}_{\Lambda}$Si, ${}^{32}_{\Lambda}$S, ${}^{33}_{\Lambda}$S, ${}^{40}_{\Lambda}$Ca, ${}^{41}_{\Lambda}$Ca, ${}^{51}_{\Lambda}$V, ${}^{56}_{\Lambda}$Fe, ${}^{89}_{\Lambda}$Y, ${}^{139}_{\Lambda}$La, and ${}^{208}_{\Lambda}$Pb. We get values of ΛN potential, $V_{\Lambda N}$

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TABLE I: A new parameterizations for Λ-N Skyrme potential derived self-consistently by fitting a large set of experimental data of Λ single particle energies for two different values of β.

SET	β	u_0 (MeV fm^3)	u_1 (MeV fm^5)	u_2 (MeV fm^5)	u_3 (MeV fm$^{3+3\beta}$)	y_0	y_3	χ^2
HP0608	1/3	-484.5329	123.8707	-2.6439	1302.5353	-0.2293	-0.3081	1.0627
HP1008	1/3	-550.0497	49.7609	-10.0550	1595.3743	-0.2999	-0.2339	1.4365
HP1808	1/3	-482.4832	87.1493	12.0396	1528.2587	-0.1177	-0.3484	1.1803
HP1908	1/3	-585.1839	134.4793	-25.4242	1011.7046	-0.4874	0.3515	1.1176
HP1308	1	-325.1456	37.9769	-15.1072	1801.6298	-0.2556	-0.0454	1.2592
HP2908	1	-524.0549	13.9880	91.8494	1708.3227	-0.7561	0.5501	1.3269
HP0909	1	-404.3997	38.2069	69.5367	1663.7979	-0.4815	0.1651	1.0747
HP1009	1	-550.5586	72.4039	67.9916	1883.1381	-0.7984	0.2417	1.2787

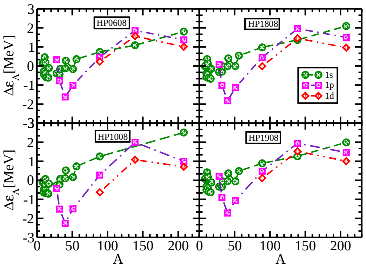

FIG. 1: The difference in the experimental and theoretical calculated single particle energies, $\Delta\epsilon_\Lambda$ plotted as a function of baryon numbers, A in hypernuclei $\beta = 1/3$.

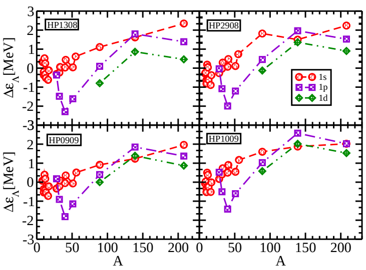

FIG. 2: Same as Fig. (1), for $\beta = 1$.

= 28.00±0.58 MeV at nuclear matter density ρ= 0.16±0.01fm^{-1} for the data of hypernuclei employed in the χ^2 minimization. The u_0 and y_0, are the parameters of zero range term, u_1 and u_2 are representing the effective and finite range term, and u_3 and y_3 are the parameters of density dependent term of the Hamiltonian density $H_{\Lambda N}(r)$ of hypernucleus. In the Figs. (1 and 2), we present the comparison of $\Delta\epsilon_\Lambda = \epsilon_\Lambda^{Expt.} - \epsilon_\Lambda^{Theor.}$ single particle energies in hypernuclei for 1s, 1p, and 1d orbitals for $\beta = 1/3$ and 1.0, respectively as a function of total baryon numbers. The relative errors in ϵ_Λ is always less than 0.1 for all the hypernuclei data employed in fit. The experimental data for binding energies $B_{\Lambda\Lambda}$ and ΛΛ interaction energy $\Delta B_{\Lambda\Lambda}$ from KEK E176 Collaboration [3] on light ΛΛ hypernuclei have been employed to search the parameterizations of effective ΛΛ interactions. In this case we get double Λ potential, $V_{\Lambda\Lambda}$ = 5.00±0.5 MeV at $\rho = 0.16$fm^{-1}.

In this work we have generated Skyrme interaction strength parameters for single Λ nucleons interaction, and double Λ nucleons effective interactions within Hartree Fock theory by using the method of successive approximations algorithm of χ^2 minimization. For the first time large set of data have been used in the fit. The parameterizations can be further employed to study the structure properties of all the available hypernuclei, to investigate bulk nuclear matter made up with hyperons at supranuclear density, to construct equation of state at zero and finite temperature of compact objects of astrophysical interest.

References

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