Neutral pion production in pp collision

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Study of pion production in pp collision has attracted considerable interest, since the total crosssection measurements for $pp \to pp\pi^0$ in the early 1990's were found to exceed the then available theoretical predictions by more than a factor of five. A recent analysis [1] of the data of Mayer et al. [2], following the model independent approach [3] and comparision with the Julich model [4] revealed that the model deviated very strongly in the case of ${}^3P_1 \to {}^3P_0p$ and to a lesser extent in ${}^3F_3 \to {}^3P_2p$, apart from highlighting the importance of the Δ contribution. This analysis took into account 12 partial wave amplitudes covering the Ss,Ps and Pp type final states. We may denote the partial wave amplitudes as $f_1, ..., f_{12}$. It was found that f_2 is dominant and its phase was chosen to be zero, since an overall phase is anyway indeterminable. The other Ps and Pp amplitudes were determined both in amplitude and phase, whereas the relative phase of f_1 remained unknown although $|f_1|$ was determined. The purpose of the present paper is to extend the model independent theoretical discussion to the spin observables in the final state and to examine how such additional experimental measurements can be used to determine empirically, the relative phase between the threshold Ss amplitude and the other near threshold amplitudes. Following [3], we write the matrix M in spin space for the reaction $pp \to pp\pi^0$ in the form

$$M = \sum_{s_i,s_f=0}^{1} \sum_{\lambda=|s_i-s_f|}^{s_i+s_f} (S^\lambda(s_f,s_i) \cdot M^\lambda(s_f,s_i)), \tag{1}$$

where s_i and s_f denote the initial and final channel spins respectively. The irreducible tensor operators $S^\lambda_\mu(s_f,s_i)$ of rank λ with μ taking values $\mu = \lambda, \lambda-1, ..., -\lambda$ are defined in [5]. The irreducible tensor amplitudes $M^\lambda_\mu(s_f,s_i)$ are expressible in terms of the 12 partial wave amplitudes and

$$A^\lambda_\mu(\mathcal{L}) = ((Y_{l_f}(\hat{\mathbf{p}}_f) \otimes Y_l(\hat{\mathbf{q}}))^{L_f} \otimes Y_{l_i}(\hat{\mathbf{p}}_i))^\lambda_\mu, \tag{2}$$

where $\mathcal{L} = \{l, l_f, L_f, l_i\}$, l being orbital angular momentum of the pion, (l_i, l_f) the initial and final relative orbital angular momenta of the two protons and L_f is the resultant of l and l_f. Thus, (2) depends purely on the angles of the pion momentum $\mathbf{q} = q\hat{\mathbf{q}}$ and $\mathbf{p_f} = p_f\hat{\mathbf{p}}_\mathbf{f} = \frac{1}{2}(\mathbf{p_1} - \mathbf{p_2})$, given in terms of the proton momenta $\mathbf{p_1}$ and $\mathbf{p_2}$ in the final state such that $\mathbf{p_1} + \mathbf{p_2} + \mathbf{q} = 0$ in the c.m. frame for the reaction. The initial c.m. momentum $\mathbf{p_i} = p_i\hat{\mathbf{p}}_\mathbf{i}$ is chosen along the z-axis. If the colliding protons are unpolarized, the spin density matrix ρ^f characterizing the two protons in the final state is given by

$$\rho^f = \tfrac{1}{4}\, M\, M^\dagger\,, \tag{3}$$

so that, the unpolarized double differential cross section may simply be written as

$$\frac{d^2\sigma_0}{dW\, d\Omega_f\, d\Omega} = Tr[\rho^f], \tag{4}$$

where W denotes the invariant mass of the two final protons. The final spin state can be completely determined empirically through experimental measurements of the polarizations

$$\mathbf{P}_i = \frac{Tr[\boldsymbol{\sigma}_i\, \rho^f]}{Tr[\rho^f]}\,, i = 1,2 \tag{5}$$

of the two protons and their spin-correlations

$$C_{\alpha\beta} = \frac{Tr[\sigma_{1\alpha}\, \sigma_{2\beta}\, \rho^f]}{Tr[\rho^f]}\,, \alpha,\beta = x,y,z \tag{6}$$

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All the above spin observables may elegantly be calculated by considering

$$P^k_\mu(s_f, s'_f) = Tr[S^k_\mu(s_f, s'_f)\,\rho^f], \qquad (7)$$

where $S^k_\mu(s_f, s'_f)$ are expressible in terms of the Pauli spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ of the two protons in the final state. Using the known properties [5]of the spin operators S^λ_μ and standard Racah techniques, we have

$$\begin{aligned} P^k_\mu(s_f, s'_f) \;&=\; \tfrac{1}{4}\sum_{s_i,\lambda,\lambda'}(-1)^{s'_f-s_i}[s'_f]\,[s_f]^2\,[\lambda][\lambda'] \\ &\times (M^\lambda(s_f, s_i)\otimes M^{\dagger\,\lambda'}(s'_f, s_i))^k_\mu \\ &\times W(s'_f, \lambda' s_f \lambda; s_i k)\,, \end{aligned} \qquad (8)$$

We focus attention on two observables which are expressible as

$$P^1_\mu(1,0) - \sqrt{3}P^1_\mu(0,1) = \frac{\sqrt{3}}{2}Tr[\rho^f\,(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)_\mu], \qquad (9)$$

and

$$P^1_\mu(1,0) + \sqrt{3}P^1_\mu(0,1) = \frac{\sqrt{3}}{\sqrt{2}}Tr[\rho^f\,(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)^1_\mu], \qquad (10)$$

which, on using (8), are expressible as

$$\begin{aligned} P^1_\mu(1,0) \mp \sqrt{3}P^1_\mu(0,1) \;&=\; \sum_{L_f=0}^{2}\sum_{l_i=1,3} \\ & C(L_f, l_i, 1; \mu, 0, \mu) \\ &\times[f_1^* F_{L_f} \pm f_1 F^*_{L_f}] \\ &\times((Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_l(\hat{\mathbf{q}}))^{L_f}_\mu, \end{aligned} \qquad (11)$$

where F_{L_f} are independent of f_1 and involve amplitudes $f_4, ..., f_{12}$, which are known from [1].
Expressing,

$$F_{L_f} = |F_{L_f}|e^{i\delta_{L_f}}, \qquad (12)$$

where both $|F_{L_f}|$ and δ_{L_f} are known from [1] and

$$f_1 = |f_1|e^{i\delta_1}, \qquad (13)$$

where $|f_1|$ alone is known and δ_1 denotes the relative phase of f_1 with respect to f_2, it follows that

$$\begin{aligned} (f_1^* F_{L_f} + f_1 F^*_{L_f}) \;&=\; 2|f_1||F_{L_f}| \\ &\times\cos(\delta_{L_f} - \delta_1) \end{aligned} \qquad (14)$$

$$\begin{aligned} (f_1^* F_{L_f} - f_1 F^*_{L_f}) \;&=\; 2|f_1||F_{L_f}| \\ &\times\sin(\delta_{L_f} - \delta_1) \end{aligned} \qquad (15)$$

will enable us to determine the relative phase δ_1 of f_1 which remained undetermined in [1].

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