Neutral pion production in $pp$ collision

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Study of pion production in $pp$ collision has attracted considerable interest, since the total cross section measurements for $pp \to pp\pi^0$ in the early 1990’s were found to exceed the then available theoretical predictions by more than a factor of five. A recent analysis [1] of the data of Mayer et al. [2], following the model independent approach [3] and comparison with the Julich model [4] revealed that the model deviated very strongly in the case of $^3P_1 \to ^3P_0pp$ and to a lesser extent in $^3F_3 \to ^3P_2p$, apart from highlighting the importance of the $\Delta$ contribution. This analysis took into account 12 partial wave amplitudes covering the $Ss$, $Ps$ and $Pp$ type final states. We may denote the partial wave amplitudes as $f_1, ..., f_{12}$. It was found that $f_2$ is dominant and its phase was chosen to be zero, since an overall phase is anyway indeterminable. The other $Ps$ and $Pp$ amplitudes were determined both in amplitude and phase, whereas the relative phase of $f_1$ remained unknown although $|f_1|$ was determined. The purpose of the present paper is to extend the model independent theoretical discussion to the spin observables in the final state and to examine how such additional experimental measurements can be used to determine empirically, the relative phase between the threshold $Ss$ amplitude and the other near threshold amplitudes. Following [3], we write the matrix $M$ in spin space for the reaction $pp \to pp\pi^0$ in the form

$$M = \sum_{s_i, s_f=0}^{1} \sum_{\lambda=|s_i-s_f|}^{|s_i+s_f|} (S^\lambda(s_f, s_i) \cdot M^\lambda(s_f, s_i)),$$

where $s_i$ and $s_f$ denote the initial and final channel spins respectively. The irreducible tensor operators $S^\lambda_{\mu}(s_f, s_i)$ of rank $\lambda$ with $\mu$ taking values $\mu = \lambda, \lambda - 1, ..., -\lambda$ are defined in [5]. The irreducible tensor amplitudes $M^\lambda_{\mu}(s_f, s_i)$ are expressible in terms of the 12 partial wave amplitudes

$$A^\lambda_{\mu}(\mathcal{L}) = \langle (Y_{l_f}(\hat{p}_f) \otimes Y_{l_i}(\hat{q}))^{L_f} \otimes Y_{l_i}(\hat{p}_i) \rangle^\lambda \mu,$$

where $\mathcal{L} = \{l, l_f, l_i, l_i\}$, $l$ being orbital angular momentum of the pion, $(l_i, l_f)$ the initial and final relative orbital angular momenta of the two protons and $L_f$ is the resultant of $l$ and $l_f$. Thus, (2) depends purely on the angles of the pion momentum $q = \hat{q} \hat{q}$ and $p_f = p_f^\perp (p_1 - p_2)$, given in terms of the proton momenta $p_1$ and $p_2$ in the final state such that $p_1 + p_2 + q = 0$ in the c.m. frame for the reaction. The initial c.m. momentum $p_i = p_i^\perp$ is chosen along the z-axis. If the colliding protons are unpolarized, the spin density matrix $\rho_f$ characterizing the two protons in the final state is given by

$$\rho_f = \frac{1}{3} M M^\dagger,$$

so that, the unpolarized double differential cross section may simply be written as

$$\frac{d^2\sigma_0}{dW \, d\Omega_f \, d\Omega} = Tr[\rho_f],$$

where $W$ denotes the invariant mass of the two final protons. The final spin state can be completely determined empirically through experimental measurements of the polarizations

$$P_i = \frac{Tr[\sigma_i \rho_f]}{Tr[\rho_f]}, i = 1, 2$$

of the two protons and their spin-correlations

$$C_{\alpha\beta} = \frac{Tr[\sigma_{1\alpha} \sigma_{2\beta} \rho_f]}{Tr[\rho_f]}, \alpha, \beta = x, y, z$$

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All the above spin observables may elegantly be calculated by considering
\[ P^k_\mu(s_f, s'_f) = Tr[S^k_\mu(s_f, s'_f) \rho_f], \]
where \( S^k_\mu(s_f, s'_f) \) are expressible in terms of the Pauli spin matrices \( \sigma_1 \) and \( \sigma_2 \) of the two protons in the final state. Using the known properties [5] of the spin operators \( S^\lambda_\mu \) and standard Racah techniques, we have
\[
P^k_\mu(s_f, s'_f) = \frac{1}{4} \sum_{s_i, \lambda, \lambda'} (-1)^{s'_f - s_i} [s'_f] [s_f]_\mu^2 [\lambda][\lambda'] \\
\times (M^\lambda(s_f, s_i) \otimes M^{\lambda'}(s'_f, s_i))_\mu^k \\
\times W(s'_f, \lambda', s_f, \lambda, s_i, k),
\]
We focus attention on two observables which are expressible as
\[ P^1_\mu(1, 0) - \sqrt{3} P^1_\mu(0, 1) = \frac{\sqrt{3}}{2} Tr[\rho_f (\sigma_1 - \sigma_2)_\mu], \]
and
\[ P^1_\mu(1, 0) + \sqrt{3} P^1_\mu(0, 1) = \frac{\sqrt{3}}{2} Tr[\rho_f (\sigma_1 \otimes \sigma_2)_\mu], \]
which, on using (8), are expressible as
\[ P^1_\mu(1, 0) - \sqrt{3} P^1_\mu(0, 1) = \sum_{L_f=0}^{2} \sum_{l_1=0}^{1} \sum_{l_2=0}^{1} L_{f}, L_{r}, l_1, l_2 \mu \\
C(L_f, l_1, 1; \mu, 0, \mu) \\
\times [f^{*}_{L_{r}, l_2} F_{L_{r}} + f_{L_{r}} F^{*}_{L_{r}}] \\
\times ((Y_{L_{r}}(\mathbf{p}_{L_{r}}) \otimes Y_{l_2}(\mathbf{q}_{L_{r}}))^L_{L_{r}},
\]
where both \( F_{L_{r}} \) and \( \delta_{L_{r}} \) are known from [1] and
\[ f_1 = |f_1| e^{i \delta_1}, \]
where \( |f_1| \) alone is known and \( \delta_1 \) denotes the relative phase of \( f_1 \) with respect to \( f_2 \), it follows that
\[
(f^{*}_{L_{r}} F_{L_{r}} + f_{L_{r}} F^{*}_{L_{r}}) = 2 |f_1| |F_{L_{r}}| \\
\cos(\delta_{L_{r}} - \delta_1)
\]
\[
(f^{*}_{L_{r}} F_{L_{r}} - f_{L_{r}} F^{*}_{L_{r}}) = 2 |f_1| |F_{L_{r}}| \\
\sin(\delta_{L_{r}} - \delta_1)
\]
will enable us to determine the relative phase \( \delta_1 \) of \( f_1 \) which remained undetermined in [1].

References