\((T - \mu_I)\) phase diagram in PNJL Model

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Introduction
Motivation behind constructing PNJL model was to unify chiral sector of NJL model [1] and confinement sector of effective Polyakov loop model. Lattice QCD (LQCD) data shows at \(\mu_q = \mu_I = 0\) chiral and deconfinement crossover coincide exactly and PNJL model gives almost this coincidence [2, 3]. Unlike non-zero \(\mu_q\) case, for \(\mu_I \neq 0\) there is no sign problem in LQCD. PNJL model in finite \(\mu_I \neq 0\) was previously studied in ref. [5–7]. In the present work we have studied \((T - \mu_I)\) phase diagram by tuning 3-momentum cutoff parameter \(\Lambda\).

Formalism
The Lagrangian of two-flavor PNJL model in finite baryon chemical potential and isospin chemical potential case, is given by

\[
\mathcal{L}_{PNJL} = \bar{\psi} \left( \gamma^\mu D^\mu + \gamma_5 \tilde{\mu} - \tilde{m}_0 \right) \psi + \frac{G}{2} \left( \bar{\psi} \psi)^2 + i \bar{\psi} \gamma_5 \tau_i \psi \right)^2 - U(\Phi[A], \overline{\Phi[A]}, T) \tag{1}
\]

where \(\psi = (\psi_u, \psi_d)^T\) is the quark field,

\[
D^\mu = \partial^\mu - iA^\mu \quad \text{and} \quad A^\mu = \delta^{\mu0}A^0 \tag{2}
\]

The two-flavor current quark mass matrix is \(\tilde{m}_0 = diag(m_u, m_d)\) and we took \(m_u = m_d \equiv m_0\). The quark chemical potential matrix \(\tilde{\mu}\) takes the form \(\tilde{\mu} = diag(\mu_u, \mu_d)\). From NJL model four-point interaction is introduced with an effective coupling strength \(G\). An effective potential \(U(\Phi, \overline{\Phi}, T)\) expressed in terms of the traced Polyakov loop \(\Phi = (\text{Tr}_c L)/N_c\) and its (charge) conjugate \(\overline{\Phi} = (\text{Tr}_c L^\dagger)/N_c\) is present in the PNJL Lagrangian. For simplicity, \(A_4\) is treated as a constant in PNJL, and the Polyakov loop is reduced as \(L = \left[ \mathcal{P} \exp \left( i \int_0^\beta A_4 dr \right) \right] = \exp \left[ iA_4 \right]\). To describe the spontaneous chiral symmetry breaking and \(I_3\) symmetry breaking, we define the chiral condensate as \(\langle \bar{\psi} \psi \rangle = \sigma\) and the pion condensates \(\langle \bar{\psi} \gamma_5 \tau_i \psi \rangle = \pi_i\), \(i=1,2,3\). where \(\tau_i\) is the Pauli matrix in flavor space.

The thermodynamical potential in the mean field level is expressed as in eqn. (3). Minimizing the thermodynamical potential (3) w.r.t. the fields \(\sigma, \pi, \Phi\) and \(\overline{\Phi}\) we obtained equilibrium field configurations. Note that when there exist multiple roots of these coupled equations, the solution corresponding to the minimal thermodynamical potential is favored.

Figures and Tables
We studied Phase diagram in \((T, \mu_I)\) plane for two cases.

- Case A : \(M = m_0 - G\sigma, \forall p\), because for finite temperature theory temperature plays the role of most natural cutoff.
- Case B : It is to some extent unphysical to admit the existence of a condensate upto arbitrarily high momentum. So we took,

\[
M = \begin{cases} 
  m_0 - G\sigma, & \text{for } p \leq \Lambda \\
  m_0, & \text{for } p > \Lambda.
\end{cases}
\]

Although case B seems to be more physical but, lattice data [8] confirmed a first order transition in high \(\mu_I\) region. We are presently investigating that by going beyond mean field if we can remove this preliminary
\[ \Omega = \frac{G}{2}(\sigma^2 + \pi^2) + \mathcal{U}(\Phi, \bar{\Phi}, T) - 2N_c \int \frac{d^3p}{(2\pi)^3} (E^-_p + E^+_p) \theta(\Lambda^2 - \hat{p}^2) \]

\[-2T \int \frac{d^3p}{(2\pi)^3} \ln \left[ 1 + 3 \left( \Phi + \bar{\Phi} e^{-\left( E^-_p - \mu \right)/T} \right) e^{-\left( E^-_p - \mu \right)/T} + e^{-3\left( E^-_p - \mu \right)/T} \right] + \ln \left[ 1 + 3 \left( \Phi + \bar{\Phi} e^{-\left( E^+_p + \mu \right)/T} \right) e^{-\left( E^+_p + \mu \right)/T} + e^{-3\left( E^+_p + \mu \right)/T} \right] + \ln \left[ 1 + 3 \left( \bar{\Phi} + \Phi e^{-\left( E^-_p - \mu \right)/T} \right) e^{-\left( E^-_p - \mu \right)/T} + e^{-3\left( E^-_p - \mu \right)/T} \right] + \ln \left[ 1 + 3 \left( \bar{\Phi} + \Phi e^{-\left( E^+_p + \mu \right)/T} \right) e^{-\left( E^+_p + \mu \right)/T} + e^{-3\left( E^+_p + \mu \right)/T} \right] \]

\[ (3) \]

FIG. 1: Comparison of phase diagrams in \((T, \mu_I)\) plane for the two cases mentioned in text.

mismatch. It should be mentioned that although \((T, \mu_I)\) phase diagram differs in these two cases, \((T, \mu_B)\) phase diagram does not depend on the mode of use of \(\Lambda\).

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References