Dual QCD and Phase Transition in Early Universe

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Introduction

The quantum chromodynamics (QCD) vacuum with condensed monopoles/dyons (i.e., a dual Ginzburg- Landau (DGL) type model of QCD or dual QCD) has been quite successful to describe the large-distance behavior of QCD vacuum [1]. Further, such DGL theory of QCD at finite temperature is also found to be useful in studying the phase transition process [2] as believed to occur in early universe.

In the present article, we have used the DGL theory of QCD with dyons to study the hadronisation in early universe. The effective potential at finite temperature is calculated by using the method as in [3]. The notions of the phase transition in the background of the dyonically condensed QCD vacuum has been investigated by calculating the critical temperature in view of the temperature dependent couplings.

Effective Potential in Dual QCD

We use the DGL model of QCD with the field–theoretic description of dyons with the following Lagrangian [4],

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)^2 + \frac{1}{2}|D_\mu \phi|^2 - V(\phi), \quad (1)$$

here $D_\mu = \partial_\mu + i\,Q\tilde{A}_\mu$ and $Q = \sqrt{e^2 + g^2}$ with e and g as electric and magnetic charge respectively. $V(\phi) = \lambda(\phi\phi^* - v^2)^2$ with $\phi = \phi_1 + i\phi_2$. The Lagrangian has U(1) symmetry therefore we consider the vacuum expectation value to be real(ϕ_o) and the thermal fluctuations in ϕ around ϕ_o by expanding it as $\phi = \phi_o + \delta\phi_1 + i\delta\phi_2$. Further, $\tilde{A}_\mu$ does not have a homogeneous mode and therefore it can also be treated as a fluctuation. It is well-known that in the broken mode of symmetry, Eq.(1) gives rise to two mass modes (i.e., the mass of the scalar and gauge fields as $m_\phi = 2\sqrt{\lambda}\phi_o$ and $m_{\tilde{A}} = Q\phi_o$ respectively). As such the QCD vacuum acquires the properties similar to that of a relativistic superconductor due to the breakdown of symmetry and leads to a dual Meissner effect.

In order to study the effect of thermal fluctuations, let us first consider, the field equation corresponding to ϕ_1 as follows,

$$\partial_\mu(\partial^\mu \phi_1) + \partial_\mu(2Q\tilde{A}^\mu \phi_2) + 4\lambda\phi_1(\phi_1^2 + \phi_2^2 - v^2) - Q^2 \tilde{A}_\mu \tilde{A}^\mu \phi_1 = 0. \quad (2)$$

On taking average of the Eq.(2), we have,

$$4\lambda(<\phi_1^3> + <\phi_2^2\phi_1> - v^2 <\phi_1>) + Q^2 <\tilde{A}_\mu \tilde{A}^\mu><\phi_1> = 0. \quad (3)$$

Now in order to quantify the thermal fluctuations, let us substitute $<\phi_1> = \phi_o$, $<\delta\phi_2> = 0$, $<\tilde{A}_\mu> = 0$ and calculate,

$$<\delta\phi_1^2> = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\varepsilon}\frac{1}{\exp(\beta\varepsilon) - 1} + (\text{T=0 term}), \quad (4)$$

where $\varepsilon = \sqrt{\vec{p}^2 + m_\phi^2}$ and $\beta = 1/kT$. Here, we absorb the zero temperature part in the definition of the renormalised zero-temperature coupling. At high temperature, up to the first order leading expansion [2],

$$<\delta\phi^2> = \frac{T^2}{12} - \frac{m_\phi T}{4\pi}. \quad (5)$$

Similarly, $<\tilde{A}_\mu \tilde{A}^\mu> = 3 <\delta\phi^2>$. The Eq.(3) can now be re-structured in terms of the above-mentioned substitutions along with Eq.(5) and $<\tilde{A}_\mu \tilde{A}^\mu>$. The integration of left hand side expressions of the restructured

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Eq.(3) *wrt* ϕ_o then gives the following form of effective potential,

$$V_{eff}(\phi_o) = \lambda\phi_o^4 - (Q^3 + 8\lambda^{3/2})\frac{T}{4\pi}\phi_o^3 + \left\{\left(\frac{2\lambda}{3} + \frac{Q^2}{8}\right)T^2 - 2\lambda v^2\right\}\phi_o^2. \quad (6)$$

Analysis of Phase Transition

Presence of cubic term of ϕ_o in Eq.(6) shows that due to thermal fluctuations the system will go through first order phase transition. Now, we consider temperature dependent expressions of λ [5] and Q [7] as $\lambda(T) = \lambda_o(1 - aT/T_c)$, where $a = 0.88$ from quenched lattice calculation [6]) and $Q^2(T) = 4\pi/[9ln(\frac{T}{0.1254T_c})^2]$ (for three quark flavors). On further analysing the Eq.(6) with the temperature dependent couplings, we obtain the transition temperature as,

$$T_c = \sqrt{\frac{0.115\lambda_o^2 v^2}{0.038\lambda_o^2 + 0.020\lambda_o - (0.027\lambda_o^{\frac{3}{2}} + 0.016)^2}} \quad (7)$$

The mass of dual gauge field in the broken phase of symmetry is $m_{\tilde{A}} = 0.192[0.333\lambda_o^{\frac{3}{2}} + 0.195]\,T_c/\lambda_o$. The higher order loop diagram contributions will be smaller and so they can be ignored. Such perturbative calculations in the present formulation are therefore valid approximations. We have plotted V_{eff} in Fig.1 as a function of ϕ_o and T using the temperature dependent expressions of couplings with $\lambda_o = 0.1$.

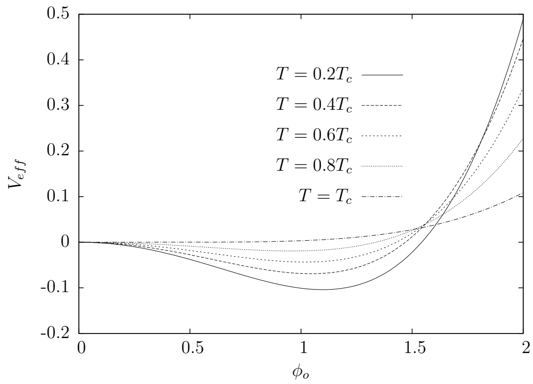

FIG. 1: Behavior of effective potential(scaled with v^4), i.e., V_{eff}/v^4 as a function of ϕ_o/v and T/v.

Conclusions

The phase transition in early universe is studied with temperature dependent coupling constants and higher order temperature corrections. It modifies the critical temperature and effective mass of gauge fields. In the framework of DGL theory, the phase transition in early universe comes to be of first order. In the DGL theory, if one incorporates the first order mass correction to both the ϕ field and the gauge field at finite temperature the perturbative study of the model is a valid approximation. Further investigations of this formulation based on the flux tube and bag model at finite temperature would be of crucial interest for much clearer understanding of the hadronisation process in early universe.

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