Relevance of Various Dirac Covariants in Hadronic Bethe-Salpeter Wave Functions in Pseudoscalar Meson Decays

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Introduction: Bethe-Salpeter Equation (BSE) is fully rooted in field theory and is a conventional non-perturbative approach in dealing with relativistic bound state problems in QCD. It provides a realistic description for analyzing inner structure of hadrons, which is also crucial in treating high energy hadronic scatterings. Despite its drawback of having to input model-dependent kernel, these studies have become an interesting topic in recent years, since calculations have shown that BSE framework using phenomenological potentials can give satisfactory results on more and more data being accumulated. Thus in this paper we study leptonic decays of pseudoscalar mesons such as π, K, D, D_s and B which proceed through the coupling of quark-antiquark loop to the axial vector current and also the two-photon decays of neutral pseudoscalar mesons such as π^0 and η_c which proceed through the famous quark-triangle diagrams. We employ QCD motivated BSE under Covariant Instantaneous Ansatz (CIA) [1,2], which is a Lorentz-invariant generalization of Instantaneous Ansatz (IA). For a $q\bar{q}$ system, the CIA formulation ensures an exact interconnection between 3D and 4D forms of BSE, where 3D BSE serves for making contact with mass spectrum of hadrons, while 4D form provides the $Hq\bar{q}$ vertex which satisfies a 4D BSE with a natural off-shell extension over the entire 4D space and thus provides a fully lorentz-invariant basis for evaluation of various transition amplitudes through various quark-loop diagrams. The BSE kernel is one-gluon-exchange type as regards colour and spin dependence and has a Lorentz-invariant confining term (see [1,2] and refs. therein for details).

Power Counting Rule: In this study, one of the main ingredients is the Bethe-Salpeter wave function (BSW). Recent studies have revealed that various covariant structures in BSWs of various hadrons are necessary to obtain quantitatively accurate observables, and that all covariants do not contribute equally and only some are relevant for calculation of meson observables. For incorporating relevant Dirac structures from their complete set in $Hq\bar{q}$ vertex, we recently proposed a power counting rule [1,2], where covariants are incorporated from their complete set for a particular meson, order-by-order in powers of inverse of meson mass, M, and thus naturally giving us a "criterion" so as to systematically choose among various Dirac covariants from their complete set to write wave functions for different mesons. For pseudoscalar meson, using this power counting rule, we can express Hadron-quark vertex $\Gamma^P(\hat{q})$ as a polynomial in various powers of $1/M$ (see Ref. [2,3] for details) as,

$$\Gamma^P(\hat{q}) = \Omega^P N_P D(\hat{q})\phi(\hat{q})/2\pi i\,; \qquad (1)$$

$$\Omega^P = \gamma_5 B_0 - i\gamma_5 \not{P} B_1/M - i\gamma_5 \not{q} B_2/M - \gamma_5[\not{P}\not{q} - \not{q}\not{P}]B_3/M^2$$

where $D(\hat{q})$ is universal denominator function, while $\phi(\hat{q})$ is the BS wave function, while $\hat{q}$ is the component of internal hadron momentum q orthogonal to total hadron momentum P, and B_i's are four dimensionless and constant coefficients (see [2-4]) to be determined. In the rest frame of hadron, we can see that each of these four terms above would receive suppression by different powers of $1/M$, implying that the maximum contribution to calculation of any meson observable should come from Leading order (LO) covariants γ_5 and $i\gamma_5\not{P}/M$ followed by next-to-leading order (NLO) covariants [2-4], $-i\gamma_5\not{q}B_2/M$ and $-\gamma_5[\not{P}\not{q} - \not{q}\not{P}]B_3/M^2$.

Leptonic and Radiative decay constants of P-mesons: Leptonic decay constants, f_P can be evaluated through loop diagram which gives coupling of two-quark loop to axial vector current as $f_P P_\mu = \langle 0|\bar{q} i\gamma_\mu\gamma_5 q|P(P)\rangle$. It has been calculated using both LO as well as NLO covariants in hadron-quark vertex, Eq.(1) (see [3] for details), and in the process we also study the relevance of both LO and NLO Dirac covariantsto f_P calculation. Decay constant f_P with full Hadron-quark vertex is then expressible in a general form as [3]: $f_P = f_P^{(0)} + f_P^{(1)} + f_P^{(2)} + f_P^{(3)}$, where $f_P^{(0)}$ and $f_P^{(1)}$ are the contributions to f_P from LO Dirac covariants γ_5 and $-i\gamma_5 \not{P}/M$ associated with constant coefficients B_0 and B_1, while $f_P^{(2)}$ and $f_P^{(3)}$ are the contributions from NLO Dirac covariants $-i\gamma_5 \not{q}/M$ and $-\gamma_5(\not{P}\not{q} - \not{q}\not{P})/M^2$ associated with constant coefficients B_2 and B_3. For details of the expression for f_P see Ref.[3]. Using the method of least square fitting of data, we find that the values of coefficients $B_0,\ldots,B_3$ (with average error with respect to experimental data < 3.5%) should respectively be [3], $B_0 = .7045, B_1 = .2626, B_2 = .0573$ and $B_3 = .0573$ to give decay constant values for five P-mesons as depicted in the Table 1.

Table1: Decay constant f_P values **(in GeV**) for π, K, D, D_s and B mesons in BSE (in last column along with the corresponding experimental data) with individual contributions from various Dirac covariants along with the contributions from LO and NLO covariants and also their % contributions for above parameter set.

	f_P^0	f_P^1	f_P^2	f_P^3	f_P^{LO}	f_P^{NLO}	f_P^{LO}(%)	f_P^{NLO}(%)	$f_P = f_P^{LO} + f_P^{NLO}$
π	.110	-.154	.000	.175	.044	.175	25%	75%	**.130**(Exp.=.130 $\pm$ .0001)
K	.202	-.104	.025	.039	.098	.064	60%	40%	**.164** (Exp.=.159 $\pm$.001)
D	.271	-.097	.010	.009	.174	.019	90%	10%	**.194** (Exp.=.220 $\pm$.020)
D_s	.426	-.156	.013	.013	.270	.026	91%	9%	**.296** (Exp.=.290 $\pm$.030)
B	.345	-.125	.005	.003	.220	.008	96%	4%	**.228** (Exp.=.240 $\pm$.040)

These findings are completely in accordance with our power counting rule proposed according to which the leading order (LO) covariants in BS wave function should contribute maximum to f_P followed by the next-to-leading order (NLO) covariants. For the lightest meson, pion the NLO contribution is quite significant due to reasons explained in [3].

To check the validity of our results, we further do calculation of radiative decay constants F_P for neutral pseudoscalar mesons such as π^0 and η_c through the process $P \to \gamma\gamma$ which proceed through the famous quark-triangle diagrams in the above framework using both the LO and the NLO covariants in the Hadron-quark vertex function, Eq.(1), taking the same values of parameters B_i 's fixed above in the calculation of f_P values of π, K, D, D_s *and* B mesons. This yields $F_\pi = .031 GeV^{-1} (Exp. = .025 GeV^{-1})$ and $F_{\eta_c} = .0067 GeV^{-1} (Exp. = .0074 GeV^{-1})$. Here, experimental values of F_P are arrived at through the expression connecting them to experimental decay widths Γ for π and η_c mesons. The numerical results for f_P and F_P obtained in our framework with use of LO and NLO covariants demonstrates the validity of our power counting rule which also provides a practical means of incorporating various Dirac covariants in BS wave function for a hadron. By this rule we get to understand relative importance of various covariants to calculation of meson observables and thus helps in improving our understanding of hadron structures.

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