Study of heavy quarkonium with energy dependent potential

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It is well known that charmonium and bottonium states can be calculated by using a nonrelativistic Schrodinger equation . The basic reasons are: 1) the mass of charm and bottom quarks is much larger than QCD scale, which makes this system free of strong normalization effects and 2) the binding energy is small compared to the mass energy ψ and ϒ states in terms of nonrelativistic $q\bar{q}$ system governed by more or less phenomenological potentials. Many different kinds of potentials have been used, ranging from simple single power law potentials] [1] to those [2] incorporating features suggested by QCD, such as a (1/r)–dependence at a short distance and /or a linear confinement potential for larger distances.

Recently new classes of potentials, namely energy dependent potentials have been used as confining potential to describe the heavy quark system. [3-4]

In the present work we have studied mass spectra of charmonium and bottonium using the following energy dependent model in the framework of nonrelativistic Schrodinger equation .

$$\left[-\frac{\hbar^2}{2\mu}\vec{\nabla^2}+V(r,E_{n,l},g)\right]\psi_{n,l}(\vec{r})=E_{n,l}\psi_{n,l}(\vec{r}) \quad (1)$$

$$\mu=\frac{m_q m_{\bar{q}}}{m_q+m_{\bar{q}}}$$

(Notation $\hbar=c=1$ is considered)
where the potential used is

$$V(r,E_{n,l},g)=\frac{1}{2}mr^2(1+\gamma E)+\frac{g}{r^2} \quad (2)$$

The first term is energy dependent harmonic oscillator and second term proportional to $1/r^2$ is added to improve the short term interaction.

The Schrodinger equation has been solved analytically and eigen values are given by this expression

$$E_{n,l}=\frac{a^2\omega^2\gamma}{8}+\frac{a\omega}{8}\sqrt{a^2\omega^2\gamma+16} \quad (3)$$

where $a=4n-2+\sqrt{(2l+1)^2+8\mu g}$

The energy eigenvalues are dependent on three parameters ω , γ and g. These parameters have been determined by fitting 2s, 3s and 1p levels and have been used to predict higher levels. The energy spectra of charmonium and bottonium system with respect to E_{1S} are shown in fig 2and 3. It also shows comparison with the experimental data and with harmonic oscillator plus g/r^2without energy dependent. Good agreement with the experimental levels has been obtained.

The resulting potentials V_1 and V_2 of charmonium and bottonium system respectively for ground state are shown in fig 1 and parameters are given in table 1. Level spacings of excited stated are shown in Table2.

Table1. Parameters calculated by equation (3)

	ω fm^{-1}	γ GeV^{-1}	g GeV^{-1}
$c\bar{c}$ (1.6GeV)	.203	-.117	-.194
$b\bar{b}$ (4.4GeV)	.187	-.102	-.64

Table 2:Saturarion property with energy dependence

	$c\bar{c}$		$b\bar{b}$	
	$\gamma=0$	$\gamma \neq 0$	$\gamma=0$	$\gamma \neq 0$
$E_{4S}-E_{1S}$ (GeV)	1.38	1.29	1.28	1.21
$E_{5S}-E_{1S}$ (GeV)	1.77	1.75	1.62	1.51
$E_{6S}-E_{1S}$ (GeV)	2.15	1.85	1.99	1.78

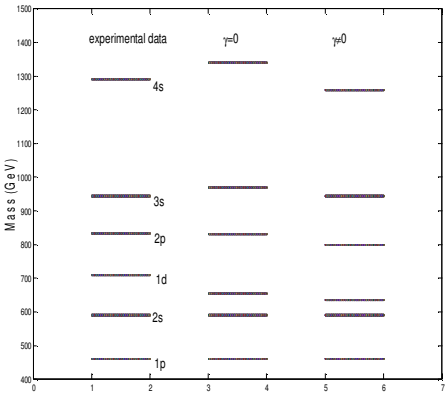

Fig 2: Mass spectrum of $c\bar{c}$ compared with experimental value.

Thus we observe that with energy dependent potential the energy level spacings for higher excited $q\bar{q}$ states shows sign of saturation.

In conclusion we can say that the new class of energy dependent potential can give good description of charmonium and bottonium properties. The energy dependence in the potential is naturally introduced since it arises from the relativistic aspect of the problem

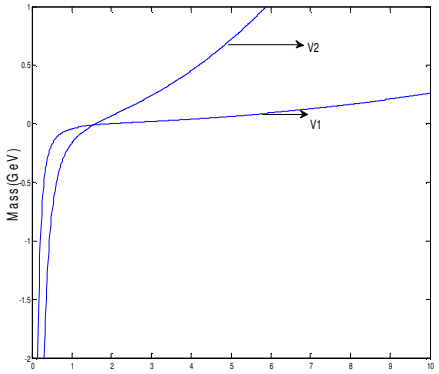

Fig 1: Potential curves for charmonium and bottonium system

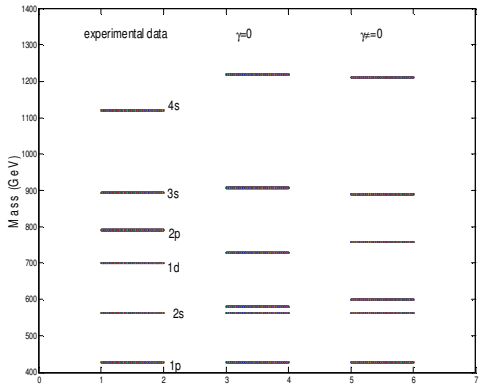

Fig 3: Mass spectrum of $b\bar{b}$ compared with experimental value.

References:

1. A. Martin: Physics Lett.B ,**93** 338 (1980); **100**, 511 (1981)
2. E. Eichen K. Gottfried,T. Kinoshita Phys. Rev. D **17** 3090 (1978)
3. R.J. Lombard , J. Mars and C. Volpe J. Phy G. Nucl. Part. Phy **34** (2007)1879-188