Study of charmed baryon $\sum_c^o(ddc)$ using hyperspherical harmonics formalism

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In the present study the bound state calculation of charmed baryon $\sum_c^o(ddc)$ is done by solving the coupled integral equations using the Hyperspherical Harmonics Expansion (HHE) method. The HHE method was developed by Simonov [1] and was used by Delves [2] to describe the three body channels in nuclear interactions. Verma and Sural [3] have also utilized HHE method for calculating the binding energy (B.E.) of hypertriton with tensor potentials in the three-body structure. The HHE method has been used to investigate the properties of a number of three-body and four-body systems.

The first L = 0 lowest state of charmed baryon $\sum_c^o(ddc)$ formed by heavy quark c (charm) has been studied. For this purpose the non-relativistic treatment is justified [4] and the HHE method is used to solve the three-body Schrödinger equation for the power law confining potential between quark pairs [5]. We have used different set of potentials and have retained a number of hyperspherical harmonics to get a convergent result for the ground state energy of three-quark bound state of charmed baryon $\sum_c^o(ddc)$.

The hypercentral potential depends only on the hyperradius which itself is a function of Jacobi coordinates that are functions of particle positions $(\vec{r}_1, \vec{r}_2, \ldots\ldots\ldots\ldots, and\, \vec{r}_N)$. The Jacobi Coordinates are defined as

$$\vec{\eta} = \sqrt{\frac{\mu_{1,2}}{\mu}}(\vec{r}_1 - \vec{r}_2);\ \vec{\xi} = \sqrt{\frac{\mu_3}{\mu}}\left(\frac{\vec{r}_1 + \vec{r}_2}{2} - \vec{r}_3\right) \quad \ldots. (1)$$

The coefficients are function of the masses.

After eliminating the centre of mass motion of the three quark system the Schrödinger equation is written in a six-dimensional space which consists of the hyperradius ρ and the five angular variables $\Omega\rho$. After eliminating the angular part $\Omega\rho$, the final Coupled Integral Equation (CIE) for the hyperradial functions $\chi_K^v(\rho)$ also called the partial waves of the system of three-particles has the form

$$\chi_K^\upsilon(\rho) = \int_0^\infty I_{K+2}(\kappa\rho_<)K_{K+2}(\kappa\rho_>) \times \sum_{K',\upsilon'} U_{KK'}^{\upsilon\upsilon'}(\rho')\chi_{K'}^{\upsilon'}(\rho')\rho' d\rho' \quad \ldots\ldots. (2)$$

Here K is the Global angular momentum quantum number and takes only even positive values and υ takes value from $-\frac{K}{2}$ to $+\frac{K}{2}$ in the steps of 2. $I_{K+2}(\kappa\rho_<)$ and $K_{K+2}(\kappa\rho_>)$ are modified Bessel's function of order K+2 and are calculated numerically using standard subroutine program. Here $\kappa = \sqrt{-2\mu E}$; E is the B.E. of the system under study. The matrix- elements $U_{KK'}^{\upsilon\upsilon'}$ are the sum of three two-body pair interaction and has the following form

$$U_{KK'}^{\upsilon\upsilon'} = \langle u_K^v | \hat{V} | u_{K'}^{v'} \rangle = -2\mu \int u_K^v \{\hat{V}^{(12)} + \hat{V}^{(13)} + \hat{V}^{(23)}\} u_{K'}^{v'} d\Omega\rho \quad \ldots\ldots. (3)$$

Ground state energy is investigated by solving Eq(1) using Gauss quadrature Formula. The number of K-harmonics to be included in the integral equation is determined by the convergence of the numerical procedure

We have chosen to study charmed baryon $\sum_c^o(ddc)$. In the present calculation the mass of down quark has been taken to be 390 Mev and that of the charm quark as 1750 Mev [6]. The mass of $\sum_c^o(ddc)$ taken is 2455 Mev as reported in ref. [6]. In the first stage we have taken Quigg & Rosner potential [7] as q-q potential to study $\sum_c^o(ddc)$. The form of the Quigg & Rosner potential is given as

$$V(r_{ij}) = A_e^{-r_{ij}^2/\beta^2} + Br_{ij}^2 + C \qquad \ldots\ldots (4)$$

Here A, B & C are constants. r_{ij} is the interparticle separation & β is the range of the potential. We have calculated the B.E. of $\sum_c^o(ddc)$ using the above form of q-q potential given by Eq (5) and the result is given in the following table -1.

Table -1 : B.E. of $\sum_c^o(ddc)$ for K = 0 harmonics

S. No.	β (fm)	A (Mev)	B (MeV –fm^{-2})	C (MeV)	B.E. (MeV)
1	1.0	-330	-4.3	29.5	75
2	0.8	-430	-4.6	24.8	75
3	0.6	-626	-4.8	16.2	75
4	0.4	-1132	-5.0	1.5	75

The B.E. of $\sum_c^o(ddc)$ given in Table – 1 for K = 0 harmonics using different sets of Quigg & Rosner potential parameters.

Table -2: $\sum_c^o(ddc)$ (% contribution of K to B.E.)

	β (fm)	B.E. (MeV)	% Cont. of different K - Harmonics		
			K=0	K=4	K=6
Set 1	1.0	75	89.94	8.01	2.05
Set 2	0.8	75	91.41	8.01	0.58
Set 3	0.6	75	88.06	11.93	0.01
Set 4	0.4	75	75.51	24.00	0.49

We observe that the major contribution to the B.E. comes from K = 0 harmonics and goes on decreasing as the range of potential decreases or the depth increases. We conclude that HH expansion converges for inclusion of harmonics upto K=6 only. It is in conformity with earlier HH expansion calculation on triton where the lowest K-harmonics gives about 97 % of the total B.E. The sets 1 and 2 (Table -2) gives a realistic value of q-q potential. Thus we can conclude that calculations using HHE technique.

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