Properties of Mesons with Generalized Pöschl-Teller Potential Using SUSYQM

Rajendra H Parmar*
Sir P T Science College,
Modasa 383 315, INDIA.
**E-mail:rajendra@scientist.com*

Study of hadron physics is intimately connected to the knowledge of strong interactions. Since the introduction of pions by by Yukawa to explain the inter-nucleon force and since the quark structure of hadrons known, our knowledge of hadrons and in parallel, our understanding of the strong interactions have undergone several major revisions. It is very imortant to study the structure and properties of hadrons in order to understand the quarks dynamics completely, which is governed by quantum chromodynamics(QCD) [1].

With acceptance of QCD as the theory of the strong interactions, comes the need to understand its physical states. Interpretation of the the spectrum of hadrons reveals information on the nonperterbative aspects of QCD. Unfortunately, calculating the properties of the hadrons from the QCD Langrangian has proven to be a very difficult task in this strongly coupled nonlinear theory. In the long term, the most promising technique is formulating the theory on a discrete spacetime lattice [2]. By constructing interpolating fields with the quantum numbers of physical hadrons and evaluating their correlation's on the lattice, one is able to calculate hadron properties from first principles. Although a great deal of progress has been made, it has been slow since these calculations take enormous of computer time. Additionally, a disadvantage of this approach is that one may obtain numerical results without any corresponding physical insight. The constituent-quark model offers the most complete description of hadron properties and is probably the most successful phenomenological model of hadron structures [1]. The methods of analysis used are very similar to those in atomic physics, where one uses either nonrelativistic or relativistic quantum mechanics.

The SUSY QM (Superesymmetric quantum mechanics) methods to solve the energy eigen values of apparantly unknown potentials are very useful. For example, the condensate mentioned here is similar to the exactly solvable potential called Pöschl-Teller potential [3]. We make an attempt to introduce the generalized Pöschl-Teller potential given by

$$V(r) = k_0 + k_1 \mathrm{cosech}^2 Cr - k_2 \coth Cr \ \mathrm{cosech} Cr \tag{1}$$

Here, super potential, $\Phi(r) = -\frac{1}{\sqrt{2\mu}} \frac{\psi_0'^{(-)}(r)}{\psi_0^{(-)}(r)} = \frac{1}{\sqrt{2\mu}}(A\ cothCr - B\ cosechCr)$, provides the Generalized Pöschl-Teller Potential V^- as[1],

$$V^{(-)} = A_1^2 + (A_1^2 + B_1^2 + A_1\ C_1)$$

$$cosech^2 Cr - B_1\ (2A_1 + C_1)\ cothCr\ cosechCr \tag{2}$$

Here, $A_1 = \frac{1}{\sqrt{2\mu}}A$, , $B_1 = \frac{1}{\sqrt{2\mu}}B$, $C_1 = \frac{1}{\sqrt{2\mu}}C$ and μ represents the reduced mass. It has the expected behaviour for static Quark-antiQuark potential[1]. The Schrödinger equation for this potential can be solved analytically using the methods of SUSYQM. The resultant radial solution for $l = 0$ states and the eigenvalues $E_n^{(-)}$ are given by[3],

$$\psi_n^{(-)}(r) = (y-1)^{\frac{\lambda-s}{2}}\ (y+1)^{-\frac{\lambda+s}{2}}$$

$$P_n^{(\lambda-s-\frac{1}{2};-\lambda-s-\frac{1}{2})}(y) \tag{3}$$

$$E_n^- = A_1^2 - (A_1 - n\ C_1)^2 = E_n - E_0 \tag{4}$$

Where, $y = coshCr$, $\lambda = B/C$, $s = A/C$ and $P_n^{(a,b)}(y)$ are the Jacobi polynomials.

TABLE I: Spin average Energy of Mesons in MeV Subtracted by its Ground state Energy: ($A = 588MeV$, $C = 103MeV$, $m_q = 300MeV$, $m_s = 450MeV$, $m_s = 1.5GeV$ and $m_b = 4.5GeV$). Bracketed quantities are from Ref[5].

State	α_s	E_{2s-1s}	E_{3s-1s}	E_{4s-1s}	E_{5s-1s}	E_{6s-1s}
$q\bar{q}$	0.700	0752	1360	1823	2142	2316
		(0829)	(1375)	(1829)	(2202)	
$q\bar{s}$	0.646	0735	1330	1784	2096	2268
		(0810)	(1342)	(1787)	(2154)	
$q\bar{c}$	0.574	0670	1211	1625	1909	2065
		(0707)	(1198)	(1610)	(1936)	
$q\bar{b}$	0.506	0768	1388	1861	2186	2364
		(1012)	(1431)	(1837)	(2107)	
$s\bar{s}$	0.581	0727	1315	1763	2071	2240
		(0770)	(1287)	(1784)	(2084)	
$s\bar{c}$	0.488	0670	1211	1625	1909	2065
		(0742)	(1211)	(1610)	(1929)	
$s\bar{b}$	0.428	0736	1331	1785	2097	2268
		(0888)	(1372)	(1756)	(1999)	
$c\bar{c}$	0.361	0584	1020	1368	1607	1737
		(0606)	(1005)	(1352)	(1634)	
$c\bar{b}$	0.298	0552	1000	1344	1582	1715
		(0651)	(1022)	(1330)	(1559)	
$b\bar{b}$	0.225	0485	0877	1177	1383	1497
		(0569)	(0872)	(1121)	(1400)	(1560)

The parameters A, B and C, depend on the Quark masses as well as the strong running coupling constant α_s as $A \sim \sqrt{2\mu}\ \alpha_s\ A_1$. For different flavour combinations, $\sqrt{2\mu}\ \alpha_s$ can be obtained from the constituent Quark masses and α_s. The spin average energy(E_n) of the mesonic systems are then obtained by realizing that $E_n^- = E_n - E_0$ in accordance with SUSYQM, where E_0 is the ground state spin average energy of the mesonic system. The results are tabulated in comparison with other theoretical calculations[5]. Other calculated parameters will be presented in the symposium.

References

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