Gluon and quark self-energy in polarized quark matter

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Introduction

Recently, the possibilities of ferromagnetic phase transition have been studied by various authors[1]. Such a phase transition might occur for strange quark system at low densities. The underlying mechanism is very similar to what happens for degenerate electron gas in the non-relativistic limit which is known as Bloch transition. In the relativistic limit such a transition might take place where the mechanism will be different. The para-ferro phase transition in quark matter can be studied by evaluating the exchange energy and by comparing the same with the kinetic energy. At low densities the former generates enough attraction to win over its kinetic counterpart. This gives rise to phase transition. However, such calculation remains incomplete because higher order terms involve infrared divergences due to the exchange of massless gluons. One, therefore, needs to evaluate the self-energies of gluons and quarks in polarized matter at finite density in order to proceed further. This is what we attempt to do here.

Formalism:

Gluon self-energy which is required to derive the correlation energy in matter with arbitrary spins, can be obtained by evaluating the diagram shown below[2],

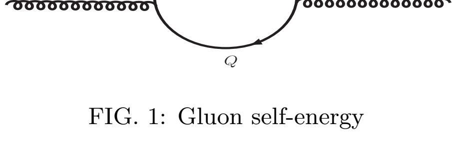

FIG. 1: Gluon self-energy

$$
\begin{aligned}
\Pi_{\mu\nu} &= \frac{N_f g^2}{2}\int\frac{d^3q}{(2\pi)^3}\sum_{s=\pm}\frac{\theta_q^s}{2E_q^s}\Big\{\frac{P^2}{P^4-4(Q.P)^2} \\
&\times \sum_{s'=\pm}[\mathcal{M}^{ss'}_{\mu\nu}(P+Q,Q)+\mathcal{M}^{ss'}_{\mu\nu}(Q,Q-P)] \\
&- \frac{2(Q.P)}{P^4-4(P.Q)^2}\sum_{s'=\pm}[\mathcal{M}^{ss'}_{\mu\nu}(P+Q,Q) \\
&- \mathcal{M}^{ss'}_{\mu\nu}(Q,Q-P)]. \qquad (1)
\end{aligned}
$$

Here $\mathcal{M}^{ss'}_{\mu\nu}$ is the Compton scattering amplitude, $\theta_q^\pm = \theta(q_f^\pm - |q|)$ and the spin projection operator $\mathcal{P}(a_s) = \frac{1}{2}(1+\gamma^5 \not{a}_s)$ is used at each vertex where $a_s = s + \frac{q(s.q)}{m_q(E_q+m_q)}$; $a_s^0 = \frac{q.s}{m_q}$.

The longitudinal and transverse gluon self-energy can be expressed as follows,

$$
\begin{aligned}
\Pi_L &= \frac{g^2}{4\pi^2}(1-C_0^2)\sum_{s=\pm} q_f^s\epsilon_f^s\Big[1-\frac{C_0}{2v_f^s}\ln\Big(\frac{C_0+v_f^s}{C_0-v_f^s}\Big)\Big] \\
\Pi_T &= \frac{g^2}{16\pi^2}C_0\sum_{s=\pm} q_f^{s2}\Big[\frac{2C_0}{v_f^s}+\Big(1-\frac{C_0^2}{v_f^{s2}}\Big) \\
&\times \ln\Big(\frac{C_0+v_f^s}{C_0-v_f^s}\Big)\Big]. \qquad (2)
\end{aligned}
$$

Where $C_0 = \frac{p_0}{|p|}$, $v_f^\pm = q_f(1\pm\xi)^{\frac{1}{3}}/(q_f^2(1\pm\xi)^{\frac{2}{3}} + m_q^2)^{\frac{1}{2}}$, $q_f^\pm = q_f(1\pm\xi)^{\frac{1}{3}}$ and $q_f = (\pi^2 n_q)^{\frac{1}{3}}$. Here total spin up and down quarks density is $n_q = n_q^+ + n_q^-$ and $\xi = (n_q^+ - n_q^-)/(n_q^+ + n_q^-)$.

Gluon dispersion relations:

Collective modes are characterized by a dispersion law. By setting transverse dielectric function equal to zero[3],

$$
\begin{aligned}
\epsilon_T = 0 &= 1+\frac{g^2}{16\pi^2}\frac{C_0}{(p^2-\omega_T^2)}\sum_{s=\pm}\Big[q_f^{s2}\Big\{\frac{2C_0}{v_f^{s2}} \\
&+ (1-\frac{C_0^2}{v_f^{s2}})\ln\Big(\frac{C_0+v_f^s}{C_0-v_f^s}\Big)\Big\}\Big] \qquad (3)
\end{aligned}
$$

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two identical transverse modes can be obtained. For $p \to 0$, $\omega_T^2 \simeq \frac{3}{2}p^2 + \frac{a}{12}$, and for $p \to \infty$, $\omega_T^2 \simeq p^2 + \frac{a}{6}$. Where $a = \frac{g^2}{\pi^2} q_f^2 [(1+\xi)^{\frac{2}{3}} + (1-\xi)^{\frac{2}{3}}]$.

The other longitudinal mode can be obtained by setting longitudinal dielectric function equal to zero,

$$\epsilon_L = 0 = 1 - \frac{g^2}{4\pi^2}\frac{1}{p^2}\sum_{s=\pm}[q_f^s \epsilon_f^s\{-1 + \frac{C_0}{2v_f^s} \times \ln\Big(\frac{C_0 + v_f^s}{C_0 - v_f^s}\Big)\}\Big]. \quad (4)$$

For $p \to 0$, $\omega_L^2 \simeq \frac{a}{12} + \frac{3}{5}p^2$, and for $p \to \infty$, $\omega_L \simeq p(1 + 2e^{-2(\frac{4\pi^2 p^2}{g^2 a}+1)})$.

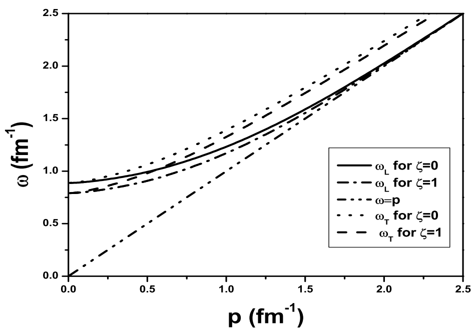

FIG. 2: Plot of gluon dispersion relations for polarized and unpolarized matter

Quark self-energy:

Like gluon, quark self-energy can also be written in terms of quark forward scattering amplitude. Fermionic propagator can be written as $G(P) = G^F(P) + G^D(P)$[4]. Considering the relevant density dependent part we can write

$$\bar{u}(P)\Sigma(P)u(P) = \frac{9\pi}{4}\int\frac{d^3q}{(2\pi)^3}\sum_{s=\pm}\frac{\theta^s(q_f - |q|)}{E_q} \times \sum_{s'=\pm}\mathcal{M}^{ss'}. \quad (5)$$

Where, $\mathcal{M}^{ss'}$ is the forward scattering amplitude of the two quarks by the one gluon exchange. Here only exchange diagram contributes. Now, the matrix amplitude can be written as follows[1]

$$\mathcal{M}^{ss'} = \sum_{ij}[\bar{u}_\beta(P)g(t^a)_{ji}\gamma_\mu\mathcal{P}(a_s)u_\alpha(P+Q)] \times \frac{g^{\mu\nu}}{Q^2}[u_\alpha(P+Q)g(t^a)_{ij}\gamma_\nu\mathcal{P}(a_{s'})u_\beta(P)] = \frac{4}{9}\frac{g^2}{Q^2}\mathrm{Tr}[\gamma_\mu\mathcal{P}(a_s)(\not{P}+\not{Q})\gamma^\mu\mathcal{P}(a_{s'})\not{P}]. \quad (6)$$

Summary and Conclusion:

In this work we evaluate gluon and quark self-energies in spin polarized quark matter. Such a system might exist in quark stars. Hence, with the results what we have presented here, one can calculate various observables like bulk energies of the system with correction due to polarization or neutrino emissivity etc. for quark matter with arbitrary spins.

References

[1] T.Tatsumi, Phys. Lett. **B489**, 280 (2000).
[2] K. Pal, S. Biswas, and A.K.Dutt-Mazumder, Phys. Rev. C**80**, 024903 (2009).
[3] S. A. Chin, Ann. Phys. **108**, 301 (1977).
[4] S. Biswas and A.K.Dutt-Mazumder, Phys. Rev. C**77**, 045201 (2008).