Multifractal moments in high-energy heavy-ion interactions

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Introduction

Fluctuations of dynamical nature were first observed in the JACEE event [1] having unusually very high multiplicity. This has generated considerable interest in the study of multiparticle production in relativistic hadronic and nuclear collisions. It has been suggested [2,3] that occurrence of large particle density fluctuations in small rapidity bins exhibit self-similarity. The method of Gq moments, defined for both positive and negative q values was suggested [4] for the study of fractal nature of particle emitting source and hence <Gq> should exhibit a power law behaviour[4-6] over a range of small δη in the following fashion:

$$\langle G_q \rangle \propto (\delta\eta)^{Tq} \qquad (1)$$

where Tq are the mass exponents and may be determined from the observed linear dependence of ln <Gq> on ln (δη). The multifractal spectrum, $f(\alpha_q)$ and the Renyi dimension, D_q are, therefore, studied to examine the fractal nature of the particle emitting source in the present analysis of the data.

Details of the data

In the present study a stack of ILFORD G5 emulsion, exposed to 14.5A GeV/c ^{28}Si nuclei has been used. Lund Model FRITIOF events with the same description have been generated and analyzed to compare with the experimental results.

Results and discussion

The Renyi dimension, D_q and multifractal spectral function, $f(\alpha_q)$ for both the experimental and FRITIOF data sets are calculated and the variations of D_q with q and $f(\alpha_q)$ versus α_q for both the data sets are exhibited in figures 1 and 2. It is clear from the figure that the Renyi dimension, D_q decreases with increase of order of the moments from -6 to +6 which indicates the presence of multifractality in multiparticle production in 14.5A GeV/c ^{28}Si nucleus collisions.

The multifractal spectrum, $f(\alpha_q)$ is lying between -2.48 and 0.71 for the experimental and -2.68 and 0.79 corresponding to FRITIOF generated data respectively. The spectra are concave downwards and having a common tangent for $f(\alpha_q) = \alpha_q$.

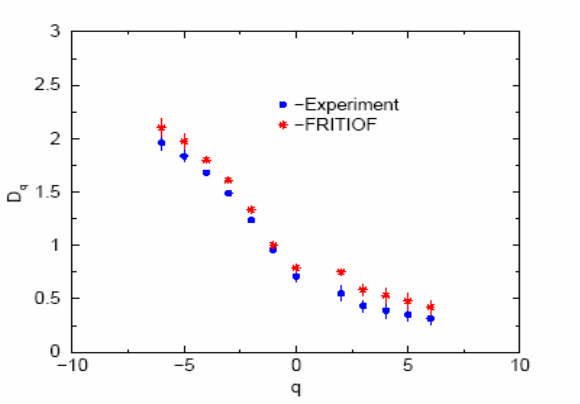

Fig. 1

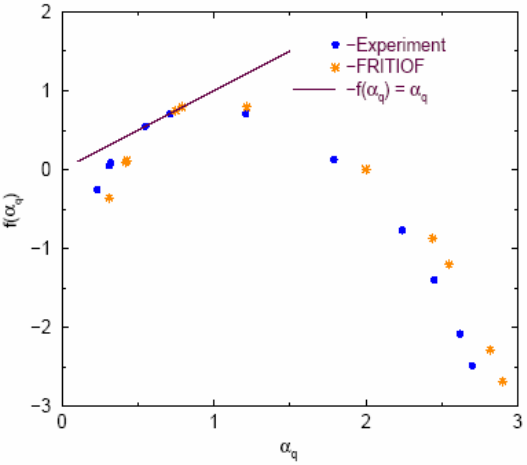

Fig. 2

In order to examine the dependence of D_q on q and $f(\alpha_q)$ on α_q on the target size, the plots have been obtained for three groups of interactions and displayed in Figs. 3 and 4. The generalized dimensions have higher values for the interactions due to heavier targets for each order of the moments. One of the reasons for the higher values of the Renyi dimension for the interactions due to heavier targets may be attributed to increase in the mean multiplicity with increasing target mass.

From the f (α_q) versus α_q plots one may conclude that:

1. In each case the multifractal spectrum, f (α_q) is represented by continuous curve, thus characterizing a quantitative manifestation of the fluctuation in the rapidity space.
2. The spectra are concave downward in shape.
3. The values of f (α_q) are relatively smaller for lighter targets and the spectrum become relatively wider with increasing target size.

The $f(\alpha_q)$ spectrum may be used to estimate the Renyi dimensions, D_q which is the basic property of any fractal measure. The values of Renyi's dimensions, D_0 , D_1 and D_2 , which are regarded very sensitive to the production mechanism, are calculated for experimental as well as FRITIOF data for 14.5A GeV/c ^{28}Si nucleus collisions. These values come out to be 0.710±0.011, 0.680±0.017 and 0.790±0.022 for experimental data and 0.790±0.022, 0.660±0.005 and 0.535±0.032 for FRITIOF data respectively.

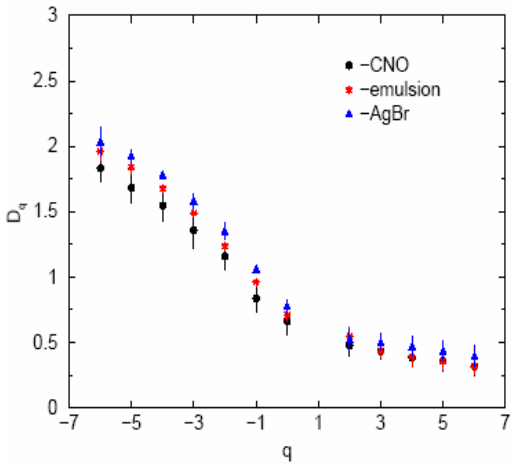

Fig. 3

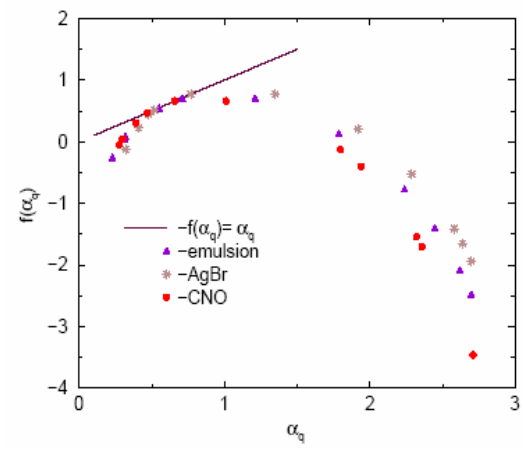

Fig. 4

Conclusions

Based on the calculations of multifractal moments, multifractal analysis in 14.5AGeV/c ^{28}Si nucleus collisions is performed. The observed behaviour of the spectral function, f (α_q) in pseudorapidity space manifests self-similarity in the mechanism of multiparticle production. The decreasing trend in the value of D_q with increasing order of the moments, q confirms the presence of multifractality. A weak target mass dependence of the parameters D_q and $f(\alpha_q)$ are observed in the considered interactions of the present work.

References

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