Study of Renyi Dimension and Multifractal Spectrum in ^{28}Si-Em Collisions at14.6A GeV

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Introduction

The ultimate aim of relativistic heavy ion experiments at AGS, CERN SPS and relativistic heavy ion collider, RHIC, at Brookhaven National Laboratory is to provide an opportunity to investigate strongly interacting matter at energy densities unprecedented in a laboratory, which ultimately gives an evidence for the existence of quark-gluon plasma (QGP). The QGP is a state of matter in which quarks and gluons are no longer confined to volumes of hadronic dimensions. In deep inelastic scattering experiments, it has already been revealed that quarks at very short distances move freely, which is referred to as the asymptotic freedom. Quantum Chromodynamics (QCD) describes the strong interactions of quarks and gluons. The experimental observation of large rapidity fluctuations [1] has provided interest and excitement about their nature and origin. Bialas and Peschanski [2] suggested a power law scaling behaviour in terms of normalized scaled factorial moments, SFMs ($\langle F_q \rangle \propto M^{\alpha q}$) on the bin size and described the phenomenon as "intermittency". The SFMs method cannot only predicts the existence of large non-statistical fluctuations but it could also investigate the pattern of fluctuations and their origin. In the present experiment two emulsion stacks exposed to 14.6A GeV/c silicon beam from Alternating Gradient Synchrotron (AGS) at BNL, New York were used to collect the data. The details may be seen elsewhere.

Methodology

In order to perform a meaningful analysis of intermittency, normalized "cumulative" variables, $X(\eta)$ and $X(\phi)$ were used to reduce the effect of non-uniformity in single charged particle distributions. The cumulative variable in the phase space (say η) is defined as:

$$X(\eta) = \int_{\eta_{min}}^{\eta} \rho(\eta')\,d\eta' \Big/ \int_{\eta_{min}}^{\eta_{max}} \rho(\eta')\,d\eta'$$

where $\rho(\eta)$ is the single particle pseudorapidity density of shower particles and $\eta_{min}(\eta_{max})$ is the minimum (maximum) value of η.

Various experimental efforts have established the existence of the empirical phenomenon of "intermittency" in multiparticle production using normalized scaled factorial moments [2].

This analysis in a single phase-space dimension in η and ϕ spaces respectively was extended to two dimensions (η,ϕ) space. In order to use above formulism in two dimensions, a rectangle in the (η,ϕ)-space was considered, which was divided into $M_{\eta,\phi}=M_\eta \times M_\phi$ bins of each size $\delta\eta\delta\phi = (\Delta\eta/M_\eta)\ (\Delta\phi M_\phi)$, where the sum now extends over M^2 bins. The pseudorapidity interval, $\Delta\eta$, is divided into M bins of uniform width

$\delta\eta = \Delta\eta M = \{X(\eta_{max}) - X(\eta_{min})\}/M$.

Results and Discussions

Renyi dimension (also known as generalized fractal dimensions), D_q, and multifractal spectrum, $f(\alpha_q)$, are often used to study the presence of multifractal structure. The generalized dimension, D_q, expressed in term of intermittency index, α_q, by given relation:

$$Dq = \{1 - (\alpha_q/(q-1))\}$$

play a significant role in fractal theory.

The relation between the spectral function, $f(\alpha_q)$, and the fractal index, τ_q, is obtained through Legendre transformation as follows:

$$f(\alpha_q) = q\ \alpha_q - \tau_q,$$

with $\alpha_q = d\tau_q/d_q$ and $df(\alpha_q) / \alpha_q = q$, where α_q is the Lipschitz-Holder exponent. The following conditions are fulfilled for the existence of multifractal structure in a data provided (i) $f(\alpha_q)$, are continuous function of α_q, (ii) $f(\alpha_q)$ must have an upward convex shape with a distinct peak at $\alpha_q = \alpha_0$ and (iii) $f(\alpha_q) < f(\alpha_0)$, for $q \neq 0$. The width of the $f(\alpha_q)$ distribution is a measure of the size of dynamical fluctuations. For a purely statistical system with absolutely no fluctuations, $f(\alpha_q) = \alpha_q = 1$ for all values of q and the function, $f(\alpha_q)$, for all values of q and the function, $f(\alpha_q)$, is a straight line parallel to the y-axis at $\alpha_q = 1$in all phase spaces.

In order to calculate τ_q, we have used the relation: $\tau_q = q - 1 - \alpha_q$, to avoid calculating τ_q from G_q -moment method, because G_q-moments are dominated by statistical fluctuations, whereas F_q-moments are free from statistical fluctuations. Now the Renyi dimension, D_q, and multifractal spectrum, $f(\alpha_q)$, are calculated in η, ϕ and ηϕ spaces corresponding to q = - 0.8 to 6.6 in step of 0.2. The variations of D_q vs. q gives decreasing trend (not shown in Fig.) in one and two-dimensions respectively for different N_s-intervals. The values of D_q so obtained decrease from 1**.153 ± 0.023 to 0.279 ± 0.006** as q increases from - 0.8 to 6.6. Thus the decreasing behaviour of the generalized fractal dimensions, D_q, with increasing order of moments, q for all the N_S intervals in ^{28}Si-Em collisions at 14.6A GeV clearly indicates the presence of multifractality for our data. The plot of $f(\alpha_q)$ as a function of α_q for different N_S intervals as well as different phase spaces are shown in Fig1. It is obvious from Fig. 1 that the $f(\alpha_q)$ are represented by continuous curves. The figure also shows a distinct peak at $\alpha_q = \alpha_0$ for all the group samples and solid line represents tangent at an angle of 45° at $\alpha_1 = f(\alpha_1)$. The left hand sides (q > 0) of the spectra $f(\alpha_q)$ are sensitive to peaks and the right hand sides (q < 0) describe the valleys of single particle η-distribution, which might be responsible for producing relativistic particles in nuclear collisions. The most basic property of any fractal measure is its dimensions and a set of conventional dimensions for q = 0, 1 and 2 are the fractal dimension, $D_0 = f(\alpha_0)$, the information dimension, $D_1 = f(\alpha_1)$ and correlation dimension, $D_2 = 2\alpha_2 - f(\alpha_2)$. The values of these dimensions in η, and ηϕ phase spaces are 0**.99 ± 0.05, 0.89 ± 0.06 and 0.96 ± 0.03**, respectively. The values of D_0, D_1 and D_2 also calculated with intermittency indices are found to be **0.96 ± 0.06, 0.88 ± 0.02 and 0.91 ± 0.05**, respectively. A consistency is found in the two values obtained by the multifractal spectrum and intermittency indices. From the discussion of the Renyi dimension, D_q, and the spectral function, $f(\alpha_q)$, it may be said that no phase transition is taking place.

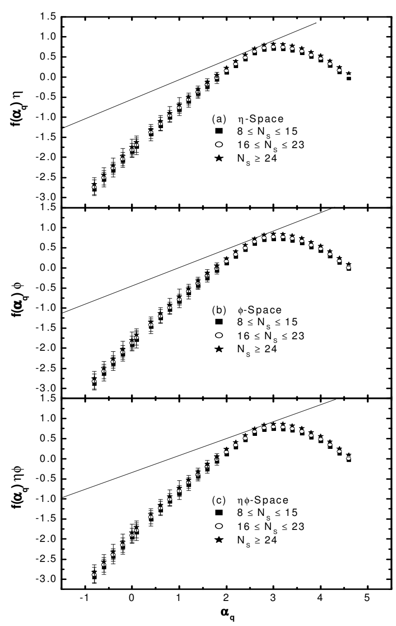

Fig.1

References

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