Correlation energy of spin polarized quark matter

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Introduction

The possibility of ferromagnetic phase transition in dense quark matter was first discussed by Tatsumi [1] where it was shown that quark liquid interacting through one gluon exchange shows spontaneous magnetic instability at low densities. In [2], we revisited this problem and have evaluated Fermi Liquid parameters for a spin polarized quark matter which were subsequently used to derive single particle spectrum and total energy density as a function of the $\xi = (n_q^+ - n_q^-)/(n_q^+ + n_q^-)$. The Fermi momenta in the spin-polarized quark matter are defined as $p_f^+ = p_f(1+\xi)^{1/3}$ and $p_f^- = p_f(1-\xi)^{1/3}$ where p_f is the Fermi momentum of the unpolarized matter ($\xi = 0$). Such an investigation was motivated by the observation of strong magnetic field in neutron star. Moreover, the theoretical conjectures about the possible existence of quark stars provide additional impetus to examine this issue further [2, 3].

In [1] and [2] calculations were restricted only to the Hartree Fock level and the higher order terms were ignored. The computation of the ground state energy, however, requires evaluation of the diagrams beyond the exchange loop *viz.* the inclusion of correlation energy as emphasized in ref.[1]. This is rather tricky as the higher order terms are plagued with infrared divergences due to the exchange of massless bosons like gluons (or photons) indicating the failure of naive perturbation theory. The problem can be cured by summing a class of diagrams which makes the perturbation series convergent and receives logarithmic corrections. In the case of degenerate electron matter this pioneering work was done by Gell-Mann and Brueckner (GB) commonly known as GB theory where the 'correlation energy (E_{corr})' of an electron gas at high density was calculated [4]. The correlation energy is actually the higher order corrections to the ground state energy beyond the exchange term in the perturbation series defined by [4]

$$E_{corr} = E - E_{ex} - E_{kin} \tag{1}$$

Here, E_{corr}, E_{ex} and E_{kin} correspond to correlation, exchange, kinetic energy density respectively. In general for electron gas interacting via. Coulomb force it takes the following form [4]

$$E_{corr} = A \ln r_s + C + \mathcal{O}(r_s) \tag{2}$$

At large Fermi momentum (p_f) i.e. in the limit $r_s = 0$, the result becomes exact. We derive here a similar expression for QCD matter [5].

Gluon self energy and correlation energy

The derivation of E_{corr} here requires the evaluation of the gluon self-energy in spin asymmetric quark matter [5]. Apart from the calculation of correlation energy, this might have applications in evaluation, for example, of the Fermi Liquid parameter (FLP) in spin polarized matter or spin susceptibility or quantities which can be expressed in terms of FLPs [2]. The detailed expression of polarization tensor $\Pi_{\mu\nu}$ are given in [5]. For brevity, we only quote longitudinal ($\Pi_L = -\Pi_{00}+\Pi_{33}$) and transverse ($\Pi_T = \Pi_{11} = \Pi_{22}$) components of the polarization tensor. The Π_L and Π_T in this limit are determined to be

$$\Pi_L = \frac{g^2}{4\pi^2}(C_0^2 - 1)\sum_{s=\pm} p_f^s \varepsilon_f^s \times \left[-1 + \frac{C_0}{2v_f^s}\ln\left(\frac{C_0 + v_f^s}{C_0 - v_f^s}\right)\right], \tag{3}$$

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$$\Pi_T = \frac{g^2}{16\pi^2} C_0 \sum_{s=\pm} {p_f^s}^2 \left[\frac{2C_0}{v_f^s} + \left(1 - \frac{C_0^2}{{v_f^s}^2}\right) \ln\left(\frac{C_0 + v_f^s}{C_0 - v_f^s}\right) \right] \quad (4)$$

Here, we take $C_0 = k_0/|k|$ and $v_f^s = p_f^s/\varepsilon_f^s$. The leading contribution to E_{corr} can be obtained by adding the contributions of ring diagrams [5, 6]:

$$E_{corr} = -\frac{i}{2} \int \frac{d^4\mathrm{K}}{(2\pi)^4} \Big([\ln(1 - D^0\Pi_L) + D^0\Pi_L] + 2[\ln(1 - D^0\Pi_T) + D^0\Pi_T] \Big). \quad (5)$$

Here D^0 is the free gluon propagator. The spatial integral of Eq.(5) can be reduced to one for the radial variable only, because all the polarization propagators are independent of the direction of three momentum transfer k. With $K_E^2 = k_0^2 + |k|^2 = -K^2$ and $\tan\theta_E = |k|/k_0$ (subscripts E implies Euclidean), Eq.(5) becomes,

$$E_{corr} = \frac{1}{(2\pi)^3} \int_0^\infty K_E^2 \mathrm{d}K_E^2 \int_0^{\pi/2} \sin^2\theta_E \mathrm{d}\theta_E \left\{ \left[\ln\left(1 + \frac{\Pi_L(K_E^2,\theta_E)}{K_E^2}\right) - \frac{\Pi_L(K_E^2,\theta_E)}{K_E^2} \right] + 2\left[\ln\left(1 + \frac{\Pi_T(K_E^2,\theta_E)}{K_E^2}\right) - \frac{\Pi_T(K_E^2,\theta_E)}{K_E^2} \right] \right\} \quad (6)$$

Following ref.[6] and performing K_E^2 integration the ring energy becomes [5]

$$E_{corr} \simeq \frac{1}{(2\pi)^3} \frac{1}{2} \int_0^{\pi/2} \sin^2\theta_E \mathrm{d}\theta_E \Big\{ \Pi_L^2 \left[\ln\left(\frac{\Pi_L}{\varepsilon_f^2}\right) - \frac{1}{2} \right] + 2\Pi_T^2 \left[\ln\left(\frac{\Pi_T}{\varepsilon_f^2}\right) - \frac{1}{2} \right] \Big\} \quad (7)$$

E_{corr} for various ξ shown in Fig.(1). This shows with increasing ξ, E_{corr} remains attractive, however, its value decreases.

Summary and Conclusion

In this work we have derived the expressions for the gluon-self energy in spin polarized quark matter which is subsequently used to calculate correlation energy. It is shown that the correlation energy for polarized quark matter is comparatively larger than the unpolarized one, although it is always attractive. We find that numerically the contribution of E_{corr} to the total energy is not very large, however, with out this the results remain incomplete because of the associated divergences of the terms beyond exchange diagrams. Furthermore, this is an important first step to include the corrections due to correlations to the spin-susceptibility or other bulk quantities of QCD matter.

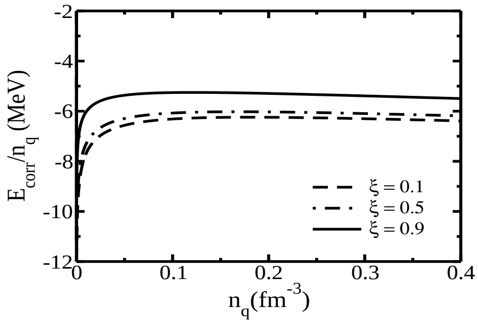

FIG. 1: Correlation energy (E_{corr}) as a function of density for different polarization parameters.

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