Spin susceptibility of dense quark matter

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Introduction

One of the active areas of high energy physics research has been exploration of the so called Quantum Chromodynamics (QCD) phase diagram. In particular, with the advent of ultrarelativistic heavy ion beams at RHIC and CERN and with the upcoming facilities of GSI where compressed baryonic matter is expected to be produced, such studies have assumed special importance. Beside the laboratory experiments, various astrophysical objects like neutron stars, quark stars, provide natural sites where many of the theoretical conjectures about the various phases of quark matter can be tested.

The original idea about para-ferro phase transition in quark matter was proposed recently in [1] where it was shown that spin polarized quark matter might exist at low density. Such phase transition, for strange quark, has also been reconfirmed in [2]. One shortcoming of these works, has been that the calculations were restricted to the Hartree Fock level and the terms beyond the exchange diagrams, commonly termed as correlation energy [3] were ignored. Without such corrections, however, the calculations are known to remain incomplete as the higher order terms are plagued with infrared divergences arising out of the exchange of massless gluons, indicating the failure of the naive perturbation series.

In the present work, we calculate the spin susceptibility (χ) of dense quark system with corrections due to correlations. This requires the knowledge of the ground state energy (GSE) of spin polarized matter with the inclusion of bubble diagrams. The GSE of the polarized quark matter has been calculated only recently in [3] which is the starting point of the present paper.

Spin susceptibility

The spin susceptibility of quark matter is determined by the change in energy of the system as its spins are polarized [4]. We introduce a polarization parameter $\xi = (n_q^+ - n_q^-)/n_q$ with the condition $0 \leq \xi \leq 1$, where n_q^+ and n_q^- correspond to densities of spin-up and spin-down quarks respectively and $n_q = n_q^+ + n_q^-$ denotes total quark density. The Fermi momenta in the spin-polarized quark matter then are $p_f^+ = p_f(1+\xi)^{1/3}$ and $p_f^- = p_f(1-\xi)^{1/3}$, where $p_f = (\pi^2 n_q)^{1/3}$, is the Fermi momentum of the unpolarized matter ($\xi = 0$). In the small ξ limit, the ground state energy behaves like [1]

$$E(\xi) = E(\xi = 0) + \frac{1}{2}\beta_s \xi^2 + \mathcal{O}(\xi^4) \quad (1)$$

Here, $\beta_s = \left.\frac{\partial^2 E}{\partial \xi^2}\right|_{\xi=0}$, defined to be the spin stiffness constant. The spin susceptibility χ is proportional to the inverse of the spin stiffness, mathematically $\chi = 2\beta_s^{-1}$.

Now, the leading contributions to the ground state energy are given by the three terms *viz.* kinetic, exchange and correlation energy density [3] i.e.

$$E = E_{kin} + E_{ex} + E_{corr} \quad (2)$$

The detailed calculation of E_{kin} and E_{ex} terms for spin polarized matter have been derived in [1, 2], which we do not discuss here. On the other hand, the expression of correlation energy E_{corr} for spin polarized matter

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have been derived in [3] which we quote here:

$$E_{corr} \simeq \frac{1}{(2\pi)^3}\frac{1}{2}\int_0^{\pi/2}\sin^2\theta_E d\theta_E\Big\{\Pi_L^2 \left[\ln\left(\frac{\Pi_L}{\varepsilon_f^2}\right)-\frac{1}{2}\right]+2\Pi_T^2\left[\ln\left(\frac{\Pi_T}{\varepsilon_f^2}\right)-\frac{1}{2}\right]\Big\} \quad (3)$$

As the spin susceptibility is given by [1]

$$\chi^{-1} = \frac{1}{2}\frac{\partial^2 E(\xi)}{\partial\xi^2}\Big|_{\xi=0}, \quad (4)$$

we have $\chi^{-1} \equiv \chi^{-1}_{kin}+\chi^{-1}_{ex}+\chi^{-1}_{corr}$. The kinetic and exchange contribution have been evaluated in ref.[1], is given by

$$\chi^{-1}_{kin} = \frac{p_f^5}{6\pi^2\varepsilon_f}. \quad (5)$$

$$\chi^{-1}_{ex} = -\frac{g^2p_f^4}{18\pi^4}\left\{2-\frac{6p_f^2}{\varepsilon_f^2}-\frac{3p_f}{\varepsilon_f^3}\left[p_f\varepsilon_f-m_q^2 \ln\left(\frac{p_f+\varepsilon_f}{m_q}\right)\right]+\frac{2p_f^2}{\varepsilon_f^2}\left[1+\frac{2m_q}{3(p_f+m_q)}\right]\right\}. \quad (6)$$

To determine the correlation correction to spin susceptibility, we expand curly braces terms of Eq.(3) in powers of the polarization parameter ξ, which gives

$$\Pi_L^2\left[\ln\left(\frac{\Pi_L}{\varepsilon_f^2}\right)-\frac{1}{2}\right]+2\Pi_T^2\left[\ln\left(\frac{\Pi_T}{\varepsilon_f^2}\right)-\frac{1}{2}\right] = (\mathcal{A}_{0L}+\mathcal{B}_{0T})+\xi^2(\mathcal{A}_{1L}+\mathcal{B}_{1T})+\mathcal{O}(\xi^4). \quad (7)$$

Here, $\mathcal{A}_{0L}$ and $\mathcal{B}_{0T}$ correspond to unpolarized matter term and the detailed expressions of $\mathcal{A}_{1L}$, $\mathcal{B}_{1T}$ are given in [7]. χ^{-1}_{corr} is

$$\chi^{-1}_{corr} = \frac{1}{2}\frac{\partial^2 E_{corr}(\xi)}{\partial\xi^2}\Big|_{\xi=0} \simeq \frac{1}{(2\pi)^3}\frac{1}{2}\int_0^{\pi/2}\sin^2\theta_E d\theta_E(\mathcal{A}_{1L}+\mathcal{B}_{1T}) \quad (8)$$

From the above expression χ^{-1}_{corr} can be estimated numerically.

In Fig.(1) we plot inverse spin susceptibility. It shows χ^{-1} changes its sign at the density $\sim 0.12\text{fm}^{-3}$ without correlation correction and when we include the correlation effect its sign changes at $\sim 0.1\text{fm}^{-3}$. It is needless to mention that this change of sign correspond to the para-ferro phase transition in dense quark system. The parameter set used here are same as those of [1–3].

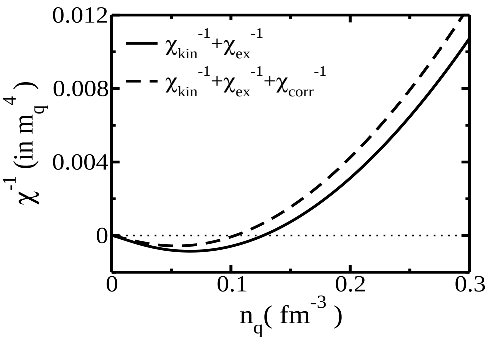

FIG. 1: Density dependence of inverse spin susceptibility.

Summary and Conclusion

In this work we have derived the spin susceptibility for degenerate quark matter with corrections due to correlation contributions. It is observed that at low density susceptibility changes sign and becomes negative suggesting the possibility of ferromagnetic phase transition.

References

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