Isobaric incompressibility of the isospin asymmetric nuclear matter with higher-order contributions

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Introduction

Isobaric incompressibility for infinite nuclear matter can be expanded in a power series as \(K_\infty(X) = K_\infty + K_s X^2 + K_t X^4 + O(X^6)\) where isospin asymmetry \(X = \rho_n - \rho_p/\rho_n + \rho_p\), \(\rho_n, \rho_p\), \(\rho = \rho_n + \rho_p\) are neutron, proton and nucleonic densities respectively. The magnitude of \(K_4\) parameter is generally quite small compared to \(K_\infty[1,5]\). The latter characterizes the isospin dependence of incompressibility at saturation density and expressed as \(K_\infty = K_{\text{sym}} - 6L - \frac{Q_\text{sym}}{K_\infty} L\) where \(L\) and \(K_{\text{sym}}\) represent, respectively, the slope and curvature parameters of the symmetry energy at the nuclear matter saturation density \(\rho_0\) while \(Q_\text{sym}\) is the third-order derivative parameter of symmetric nuclear matter (SNM) at \(\rho_0\). We study here \(K_\infty, L\) and \(K_t\).

Formulation

The nuclear matter EoS is calculated [2] using the isoscalar and isovector components of M3Y interaction with density dependence. Density dependence of this DDM3Y effective interaction is completely determined from nuclear matter calculations. Equilibrium density of nuclear matter is determined by minimizing the energy per nucleon. In a Fermi gas model of interacting nucleons, the energy per nucleon for isospin asymmetric nuclear matter [2] is:

\[
\epsilon(\rho, X) = \left[ \frac{3\hbar^2 k_F^2}{10m^2} \right] F(X) + \frac{\rho_n + \rho_p}{2} \left( 1 - \beta \rho_0 \right) \tag{1}
\]

where \(k_F = (1.5\pi^2 \rho)^{\frac{1}{3}}\) which equals the Fermi momentum in case of the SNM, \(F(X) = \frac{(1+X)^{5/3} + (1-X)^{5/3}}{2}\), the kinetic energy per nucleon \(\epsilon_{\text{kin}} = \frac{3\hbar^2 k_F^2}{10m} \epsilon_{\text{kin}}(X)\) and \(J_\text{FV} = J_{\epsilon, 00} + X^2 J_{\epsilon, 01}, J_{\epsilon, 00}\) and \(J_{\epsilon, 01}\) represent the volume integrals of the isoscalar and the isovector parts of the M3Y interaction.

Calculation and results

Third-order density derivative parameter [1] is given by \(Q_0 = 27\rho_0 \frac{\partial^3 \epsilon(\rho, 0)}{\partial \rho^3} \bigg|_{\rho=\rho_0}\). Eq.(1) says

\[
\left. \frac{\partial^3 \epsilon(\rho, 0)}{\partial \rho^3} \right|_{\rho=\rho_0} = -\frac{C J_{\epsilon, 00} (\epsilon_{\text{kin}}(0)) n(n+1)(n-1) \beta \rho_0^{n-2}}{2} + \frac{8}{45} \frac{E_F^0}{\rho_0^5} + \frac{3\lambda J_{\epsilon, 00} C}{5} n(n+1) \beta \rho_0^{n-1} \frac{E_F^0}{\rho_0} + \frac{\alpha J_{\epsilon, 00} C}{5} \times \left[ 1 - (n+1) \beta \rho_0 \right] \frac{E_F^0}{\rho_0} - \frac{4\lambda J_{\epsilon, 00} C}{45} \frac{E_F^0}{\rho_0} \right] \tag{2}
\]

where \(E_F^0 = \frac{\hbar^2 k_F^2}{2m}\) is Fermi energy for the SNM ground state, \(k_F = (1.5\pi^2 \rho_0)^{\frac{1}{3}}\), \(\epsilon_{\text{kin}}(0)\) is the kinetic energy part of the saturation energy per nucleon \(\epsilon_0\). In Table-1 of Refs.[3,4], values of approximate expression \(K_{\text{asy}} \approx K_{\text{sym}} - 6L\) for \(K_t\) are listed. The calculations are performed using the values of saturation density \(\rho_0 = 0.1533\) fm\(^{-3}\), saturation energy per nucleon \(\epsilon_0 = -15.26\) MeV for SNM and \(n = \frac{5}{3}\) [3]. Collisions involving \(^{122}\)Sn and \(^{124}\)Sn nuclei can be simulated with an improved quantum molecular dynamics transport model to reproduce isospin diffusion data from two different observables and the ratios of neutron and proton spectra. Constraints on density dependence of symmetry energy at subnormal density can be obtained by comparing these data to calculations performed over a range of symmetry energies at saturation density and different representations of the density dependence of symmetry energy [5]. Our results for

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TABLE I: Comparison of results of present calculations (DDM3Y) for various coefficients (all in MeV) characterizing isospin dependent properties of nuclear matter with those obtained by RMF models [6].

<table>
<thead>
<tr>
<th>Model</th>
<th>$K_\infty$</th>
<th>$E_{sym}(\rho_0)$</th>
<th>$L$</th>
<th>$K_{sym}$</th>
<th>$K_{asy}$</th>
<th>$Q_0$</th>
<th>$K_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>274.7 ± 7.4</td>
<td>30.71 ± 0.26</td>
<td>45.11 ± 0.02</td>
<td>-183.7 ± 3.6</td>
<td>-454.4 ± 3.5</td>
<td>-276.5 ± 10.5</td>
<td>-408.97 ± 3.01</td>
</tr>
<tr>
<td>FSUGold</td>
<td>230.0</td>
<td>32.59</td>
<td>60.5</td>
<td>-51.3</td>
<td>-414.3</td>
<td>-523.4</td>
<td>-276.77</td>
</tr>
<tr>
<td>NL3</td>
<td>271.5</td>
<td>37.29</td>
<td>118.2</td>
<td>+100.9</td>
<td>-608.3</td>
<td>+204.2</td>
<td>-697.36</td>
</tr>
<tr>
<td>Hybrid</td>
<td>230.0</td>
<td>37.30</td>
<td>118.6</td>
<td>+110.9</td>
<td>-600.7</td>
<td>-71.5</td>
<td>-563.86</td>
</tr>
</tbody>
</table>

$L$, $E_{sym}(\rho_0)$, density dependence of $E_{sym}(\rho)$ [3] are consistent with these constraints [5].

In Table-1, the values of $L$, $E_{sym}(\rho_0)$, $K_{sym}$ and $K_\tau$ obtained using exact expression $K_\tau=K_{sym}-6L\frac{\rho_0}{K_{sym}}L$ and its approximate form $K_{asy}\approx K_{sym}-6L$ are listed and compared with the corresponding quantities obtained with relativistic mean field (RMF) models [6].

In Fig.1, $K_\tau$ is plotted against $K_\infty$ for the present calculation using DDM3Y interaction and compared with the predictions of FSUGold, NL3, Hybrid [6], SkI3, SkI4, SLy4, SkM, SkM*, NLSH, TM1, TM2, DDME1 and DDME2 as given in Table-1 of Ref.[7]. The dotted rectangular region encompasses the recent accepted values of $K_\infty=250-270$ MeV [8] and $K_\tau=-370\pm 120$ MeV [1]. Although DDM3Y and SkI3 are within the above region, $L$ value for SkI3 is 100.49 MeV which is much above the acceptable limit of 45-75 MeV [9].

**Summary and conclusion**

We conclude that the approximate expression $K_{asy}\approx K_{sym}-6L$ which is quite often used in place of $K_\tau=K_{asy}-\frac{Q_0}{K_{sym}}L$ can lead to a difference of about 10% (DDM3Y) or more (FSUGold) in $K_\tau$. Present value of $K_\tau=-408.97\pm 3.01$ MeV is in reasonably close agreement with the extracted value of -550±100 MeV [10] from GMR of nuclei as light as Sn isotopes while it is in excellent agreement with the $K_\tau$ values of -389±12 MeV (NL3), -395±13 MeV (SIGO-c) [8] when extracted reproducing GMR energies of $^{208}$Pb, Sn isotopes and $^{90}$Zr among others.

**References**