Geodesic of neutrinos inside compact stars

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Introduction

Gamma Ray Bursts (GRB), and its possible connection with neutrino production in compact stars is a field of high current interest. The central engine of GRBs is still shrouded in mist, although several models has been proposed. The light curves and polarization of gamma ray burst afterglows indicate anisotropic central engines. These observations have been considered as evidence that GRBs are highly beamed. In recent times, it has been argued that most GRBs are associated with supernovae and huge neutrino production, [1] and thus, efforts are on to arrive at some picture for the GRB engine consistent with the supernovae explosion. One of the candidate for GRB model is based on the fireball model [2]. On the other hand Berezhiani *et al.* [3] have suggested that the origin of the GRBs may be associated with the deconfinement transition inside a neutron star resulting into a hybrid star (HS) or a pure quark star (QS). In reference [4], it has been shown that their is a huge amount of neutrino-antineutrino pair generation during the neutron star (NS) to strange star (SS) conversion process.

In their model during the two-flavour to three-flavour conversion, there is a huge number of neutrino-antineutrino production when excess of down quarks gets converted strange quarks. These neutrinos (and antineutrinos) could annihilate and give rise to electron-positron pairs through the reaction $\nu\bar{\nu} \to e^+e^-$. These e^+e^- pairs may further give rise to gamma rays which may provide a possible explanation of the observed GRB. Furthermore, the rotating neutron star has been shown [5] to produce the observed beaming effect. At present, it is necessary to have a better understanding of the energy deposition in the neutrino-antineutrino annihilation to electron-positron pairs in the realistic neutron star environment. Therefore the path of neutrino (or generally of massless particle) is of immense importance and needs a detailed study.

Geodesic of neutrino and MPR

The metric describing the star is the Cook-Shapiro-Teukolsky metric [6]

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{\gamma-\rho}r^2 sin^2\theta(d\phi - \omega dt)^2. \quad (1)$$

The structure of the star is obtained solving the **rns** code using tabulated equations of state (EOS). The coordinate system is oriented such that the equatorial plane lies along $\theta = \frac{\pi}{2}$ and the polar plane along $\theta = 0$. The calculation is done along these two planes. The metric is independent of $'t'$ and $'\phi'$, the coordinates are cyclic, hence the corresponding covariant generalized momenta is constant

$$p_t = p_0 = const. = -E$$
$$p_\phi = p_3 = const = L \quad (2)$$

where E and L are the total energy and total angular momentum of the particle.

Starting with magnitude of the 4 vector energy momentum is given by,

$$g_{\mu\nu}p^\mu p^\nu + {\mu_m}^2 = 0, \quad (3)$$

where μ_m is the rest mass of the particle and $p^\mu = \frac{dx^\mu}{d\lambda}$, λ being the affine parameter. The lagrangian of the system we are considering is

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given by

$$L_0 = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \quad (4)$$

where, $\dot{x}^\mu = \frac{dx^\mu}{d\lambda}$. From the Lagrangian, calculating the equation of motion for θ, it is very easy to find that it is zero along the equatorial and polar plane. Therefore the particle has at start, and continues to have $p^\theta = p^2 = 0$ in the given planes.

We define $\overline{E} = \frac{E}{\mu_m}$ and $\overline{L} = \frac{L}{\mu_m}$. As the particle is massless (neutrino), therefore we define

$$Lim_{\mu_m \to 0} \frac{\overline{L}}{\overline{E}} = b \quad (5)$$

where b is the impact parameter. Eqn 3. can be written in terms of the impact parameter. The angle θ_r between the particle trajectory and the tangent vector to the orbit can be derived by constructing the local lorentz tetrad $k^\mu{}_\nu$ for the metric. Writing eqn 3. explicitly in terms of the gravitational potentials, θ_r and b, we get

$$e^{2\alpha}[\frac{A(r,\theta)}{A(r,\theta)+\omega}]^2 \frac{e^{(\gamma-\rho)} r^2 sin^2\theta}{e^{2\alpha}} tan^2\theta_r [$$
$$\omega(1-\omega b) + \frac{e^{2\rho} b}{r^2 sin^2\theta}]^2 - e^{(\gamma+\rho)}(1-\omega b)^2$$
$$+ \frac{b^2}{r^2 sin^2\theta} e^{(\gamma+3\rho)} = 0. \quad (6)$$

This the general geodesic equation of a neutrino inside a compact star. This equation can be solved using the potentials obtained from **rns** code to obtain a minimum radius $r = R$, the minimum photosphere radius (MPR), below which a massless particle (neutrino) emitted tangentially to the stellar surface ($\theta_R = 0$) would be gravitationally bound.

The calculation is done both taking hadronic and quark EOS for a central density of 6 times nuclear saturation density. Two different rotational velocity of the stars are taken into account. The MPR is found to be greater along the polar direction than along the equatorial direction. The MPR is much greater for the hadronic star than that of the quark star. As the rotational velocity decreases the MPR increases and is maximum for the static star.

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