Nuclear matter equation of state and neutron star oscillations

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1. Introduction

Neutron stars are born when a massive star runs out of nuclear fuel and undergoes a supernova explosion in which the core of the star collapses to form a compact object, containing about one and a half times of solar mass inside a sphere of radius about ten kilometers [1]. Due to various perturbations, radial and non-radial waves travel through the star interior, that make the surface oscillate. The non-radial oscillations of neutron stars make them promising sources of detectable gravitational waves. The structure and oscillation modes of neutron stars are governed by the equation of state of the hadronic matter. The study of the vibrational motion of neutron stars has become an important tool to constrain the equation of state of nuclear matter. In section (2) we present the formalism of calculating normal mode frequencies of non-radial oscillations, and in (3) the results.

2. Formulation

The bulk motions of nuclear matter can be described in terms of the basic variables of the hydrodynamics, namely, the bulk density, ρ, the mean velocity, V_i, and the stress tensor P_{ij}. The governing equations are written as [2],

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial V_i}{\partial x_i} = 0\,, \tag{1}$$

$$\rho \frac{\partial V_i}{\partial t} + \frac{\partial P_{ik}}{\partial x_k} + \rho \frac{\partial U}{\partial x_i} = 0\,, \tag{2}$$

$$\frac{\partial P_{ij}}{\partial t} + P_{ik}\frac{\partial V_j}{\partial x_k} + P_{jk}\frac{\partial V_i}{\partial x_k} + P_{ij}\frac{\partial V_k}{\partial x_k} = 0\,, \tag{3}$$

where U is the internal potential energy. Assuming that the neutron star is a gravitationally bound nuclear matter of uniform density and the local equilibrium and large-scale motions of neutron star are dominated by newtonian gravity, the potential energy U can be known from Poisson's equation,

$$\nabla^2 U = 4\pi G \rho_0\,, \tag{4}$$

where G is the gravitational constant and ρ_0 is the density. The gravitational potential, in equilibrium, can be written for inside and outside region of the star as,

$$U_0^{in}(r) = \frac{2\pi G\rho_0}{3}\left(r^2 - 3R^2\right),\ r \leq R, \tag{5}$$

$$U_0^{out}(r) = \frac{4\pi G\rho_0 R^3}{3\,r},\quad r > R, \tag{6}$$

where R is the radius of the star. Considering the distribution of the stresses inside the star to be isotropic and the surface to be without stress,

$$P_{ij} = \delta_{ij} P_0(r)\,, \qquad P_0(R) = 0\,, \tag{7}$$

the local equilibrium pressure, $P_0(r)$, can be determined by the equation,

$$\nabla P_0(r) = -\rho_0 \nabla U_0^{in}\,, \tag{8}$$

the solution of which gives,

$$P_0(r) = -\frac{2}{3}\pi G \rho_0^2 r^2 + P_F, \tag{9}$$

where P_F is the pressure at core. Radius of neutron star can be obtained from (7),

$$P_0(R) = 0, \quad \Rightarrow R = \frac{1}{\rho_0}\sqrt{\frac{3P_F}{2\pi G}}\,, \tag{10}$$

consequently mass of the star can be found as,

$$M = \frac{4}{3}\pi R^3 \rho_0 = \frac{4\pi}{3\rho_0}\left(\frac{3P_F}{2\pi G}\right)^{\frac{3}{2}}\,, \tag{11}$$

where the core pressure, P_F, can be known from the equation of state given the average density ρ_0 which is the only parameter of this model.

To calculate the fundamental nodes of oscillations of highly incompressible nuclear matter, variational method [3] can be applied to rewrite the equations (1) - (4) as,

$$\frac{\partial \delta V_i}{\partial x_i} = 0, \tag{12}$$

$$\rho_0 \frac{\partial \delta V_i}{\partial t} + \frac{\partial \delta P_{ik}}{\partial x_k} + \rho_0 \frac{\partial \delta U}{\partial x_i} = 0, \tag{13}$$

$$\frac{\partial \delta P_{ij}}{\partial t} + P_0 \left(\frac{\partial \delta V_i}{\partial x_k} + \frac{\partial \delta V_j}{\partial x_i} \right) + \delta_{ij} \left(\delta V_k \frac{\partial P_0}{\partial x_k} \right) = 0, \tag{14}$$

$$\nabla^2 \delta U = 0, \tag{15}$$

where δV_i is the perturbed velocity of collective flow and δP_{ij} is small fluctuation in the stress tensor. Multiplying (13) by δV_i and integrating over the volume gives the energy balance equation as,

$$\frac{\partial}{\partial t} \int \left(\frac{\rho_o V_i^2}{2} \right) d\tau = \int \delta P_{ij} \frac{\partial \delta V_i}{\partial x_j} d\tau - \oint_s \left[\rho_0 \delta U \delta V_i + \delta P_{ij} \delta V_j \right] d\sigma_i \,, \tag{16}$$

We represent δV_i, and δU in the form,

$$\delta V_i = \xi_i^L(\vec{r})\, \dot{\alpha}_L(t) \,, \quad \delta U = \phi^L(\vec{r})\, \alpha_L(t) \,, \tag{17}$$

where L denotes the multipole order. The amplitude, α_L, gives spheroidal deformations of the star surface, and $\xi_i^L(\vec{r})$ is the instantaneous displacements given by

$$\xi_i^L = \frac{1}{L\, R^{L-2}} \frac{d}{dx_i} r^L P_L(\cos\theta) \,, \tag{18}$$

and the function ϕ_L on the star's surface has the form [3],

$$\phi^L = -\frac{4\pi G \rho_0 R^2}{2L+1} P_L(\cos\theta). \tag{19}$$

Inserting (17) into (14), the fluctuations in the stress tensor can be written as,

$$\delta P_{ij}\,(\vec{r}, t) = -\left[P_0 \left(\frac{\partial \xi_i^L}{\partial x_j} + \frac{\partial \xi_j^L}{\partial x_i} \right) + \delta_{ij} \left(\xi_k^L \frac{\partial P_0}{\partial x_k} \right) \right] \alpha_L \,. \tag{20}$$

Substituting (17) and (20) in (16), we get the equation of normal vibrations,

$$M_L \ddot{\alpha}_L(t) + K_L \alpha_L(t) = 0 \,, \tag{21}$$

where M_L and K_L are inertia and stiffness parameters respectively. The fundamental frequencies, $\omega_L^2 = K_L/M_L$, of the non-radial vibrations of a neutron star can be computed as,

$$\omega_L^2 = \omega_F^2\, 2(2L+1)(L-1) - \omega_G^2 \frac{2L(L-1)}{2L+1} \left[\frac{4L^2-1}{2L} - 1 \right] , \tag{22}$$

where $\omega_F^2 = P_F/(\rho_0\, R^2)$ and $\omega_G^2 = 4\pi G \rho_0/3$. The basic frequency of quantum-elastic vibrations, ω_F, depends on incompressibility, density and radius of the star, whereas and gravitation-elastic vibrations ω_G depends on the density alone.

3. Results

We take the equation of state of nuclear matter from [4] where it is calculated in the framework of non-relativistic Brueckner formalism using Bonn potential and medium modification of meson parameters. In the following table we present numerical estimates for the mass M (normalized to the solar mass, $M_\odot$), the radius R (in Km) of a neutron star and frequencies (in $10^4\,Hz$) of quadrupole(ω_2), octupole(ω_3) and hexadecapole(ω_4) vibrations corresponding to the average density ρ_0 (in fractions of the normal nuclear density ρ_N).

ρ_0	M	R	ω_2	ω_3	ω_4
1.1	1.05	11.75	1.51	2.20	2.75
1.3	1.13	11.43	1.61	2.32	2.94
1.5	1.22	11.13	1.81	2.60	3.18
2.0	1.40	10.58	2.05	2.95	3.65
2.5	1.57	10.02	2.34	3.35	4.11

References

[1] Physics of Neutron Star Interiors, ed. D. Blaschke, N. K. Glendenning, A. Sedrakian, Lecture Notes in Physics 578 (Spring-Verlag, Berlin, 2001).

[2] S. I. Bastrukov, Phys. Rev. **E 49**,1 (1994)

[3] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability (Clarendon,1961)

[4] A.B. Santra, U. Lombardo; Int J Mod Phys **E 18**,1191(2009)