Statistical entropy of a nuclear spectrometer vis-à-vis communication channel

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Introduction

The aim of this paper is to present an esoteric model of a nuclear spectrometer e.g. a gamma ray spectrometer as a communication channel [1-5]. The source entropy, receiver entropy, and joint entropy of a gamma ray spectrometer were estimated for an observed 1K gamma spectrum containing a 662keV peak from a ^{137}Cs source. The information loss estimated for the observed gamma spectrum was of the order of 94.5%. In a typical communication engineering channel, the information loss is of the order of 30% [6].

The information loss in a gamma spectrometer is far more than that in a communication channel. Hence the information extraction in a nuclear spectrometer is extremely challenging vis-à-vis communication channel. This also explains high redundancy of spectral channels, and justifies that *a priori* information required is much more than the *a posteriori* information extracted in nuclear spectrometers. Also the model amply justifies wide ranging results for the IAEA inter-comparison of spectral analysis programs in which 212 results from 163 laboratories from 34 countries were compared [7].

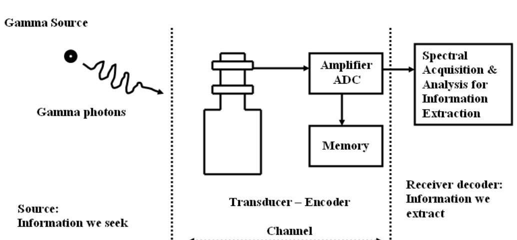

Fig. 1.Information flow in gamma spectrometer

Model of a Gamma Spectrometer

The information flow in a gamma spectrometer is depicted in Fig. 1. A source emits gamma photons at specific energies and rate, the HPGe detector and a nuclear analog to digital converter (NADC) encodes the messages, and they are acquired in a multichannel analyzer (MCA). The energy loss processes of gamma photons in a detector are by photoelectric effect, Compton scattering, and pair production through annihilation of positron and electron. The HPGe detector has Gaussian response function. The convolution of the fast electron spectra with the response function of the detector results in a nuclear spectrum. A spectral analysis program extracts the information received.

From communication engineering point of view, nuclear source emits photons with discrete energies (discrete symbols), detector and NADC act as transducer-encoder (communication channel), and MCA acts as a information extractor (decoder). For an ideal transducer encoder combination, there should be unique relationship between messages and codes generated like in a noise free communication channel. However a nuclear detector is a worst kind of transducer. A monochromatic gamma source gives rise to codes of various types. Only part of time, correct codes are generated. To make the situation worse, detector response adds to further degradation. A wrong message means encoding in a wrong spectral channel. Hence the detection processes and response function from a communication system point of degrades signal to noise ratio. This can be described quantitatively by entropy modeling of a nuclear spectrometer.

The information content or source entropy, $H(S)$, of a radiation source is:

$$H(S) = -\sum_{i=1}^{N_s} \mathrm{p}_\mathrm{i} \log_2 p_i$$

N_s is the number of different energies of the source, p_i is the probability of occurrence of an

i^{th} energy. The $H(R)$ and $H(S,R)$ are the receiver entropy and joint entropy respectively.

$$H(S,R) = H(S) + H(R/S)$$
$$= H(R) + H(S/R)$$

where H (S|R) and H (R|S) are conditional entropies when receiver and source entropies are known respectively. For an ideal, noise free, transducer, source and receiver entropies are equal and conditional entropies are zero.

The percentage information loss IL, is given by:

$$IL = 100\frac{H(R/S)}{H(S)}$$

Experimental Result

Fig. 2 depicts an observed 1K gamma spectrum

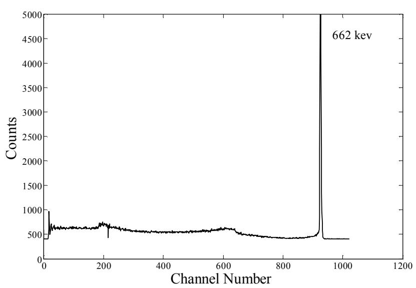

Fig. 2 Observed spectrum

containing a 662keV peak from a ^{137}Cs source. The source entropy, $H(S)$, is given as:

$$H(S) = 10 \text{ bits/energy}$$

In the Fig. 2, the maximum spectral peak counts, 6949, occur at channel number 925 while the total spectral counts are 165147. Therefore the discrete probability estimate, p_i, for correct encoding was 0.042077, and for aggregate incorrect encoding in all other spectral channels was 0.957923. To estimate the entropy value, we assumed that the probability for incorrect encoding for the remaining 1023 channels was same in the spectrum shown in Fig. 2. This approximation is justified as the spectrum shows a single energy peak of 662 keV with a small and almost flat background. Hence the probability of incorrect coding for a channel was 0.0009363861. Substituting these values, it was estimated that the information loss, IL, was 94.45% for the observed spectrum.

It is generalized that for all nuclear spectrometers the information loss is significant compared to a communication channel.

Conclusion

This paper presents an esoteric model of a nuclear spectrometer based on statistical entropy. A nuclear spectrometer is modeled as a communication system. The information loss in a gamma spectrometer is far more than that in a communication channel. Hence information extraction by a gamma spectrometer is difficult compared to that in a wireless communication system. Also the entropy modeling explains redundancy of spectral channels, and amply justifies wide ranging results for the IAEA inter-comparison of spectral analysis programs from 163 laboratories from IAEA member states.

References

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