Semiclassics of the triaxial plus particle model and chiral twins

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1. Introduction

An axial symmetry prevails in almost all the nuclei in their ground states and there is no clear evidence of triaxiality in these states. The deviation of the nuclear shape from the axial symmetry is expected at high angular momentum. The tendency of the high-j quasi-particle orbits from the unique parity sub-shell to drive the rotating nucleus towards a triaxial shape has been observed [1]. In the recent experimental data on high spin states, the signature dependence of energy, the electric quadrupole and magnetic dipole transitions are used for confirming the signature of triaxiality [2]. The most dramatic consequences of triaxial shape are the observation of two novel types of rotational bands, namely, the magnetic rotation (MR) and the chiral twins.
Chiral symmetry breaking in nuclei giving rise to nearly two degenerate ΔI=1 bands of same parity was reported first in ^{134}Pr [3].The calculations based on 3-dimensional TAC model using $\pi h_{\frac{11}{2}} \otimes \nu h_{\frac{11}{2}}$ configuration strongly suggested the chiral character in ^{134}Pr for the first time [4]. In our earlier papers [5, 6], we had presented a discussion on non-linear dynamical regime of the cranking model and axially symmetric particle-rotor model (PRM) for high-j orbital and its semiclassical quantization (SCQ) around the various fixed points. It was pointed out that the SCQ around the metastable fixed points $c_\pm$ supports a band, whose features are similar to the superdeformed band [6, 7]. In the present contribution, we have extended this semiclassical treatment to the triaxial particle plus rotor model (TPRM) with single-j shell configuration. Our dynamical results clearly show a breakdown of PT symmetry, which generate two spectra. The features of these spectra resemble very closely with those of chiral twins. The ensuing numerical results are then compared with experimental data of the chiral twins in odd-A ^{135}Nd nucleus.
Semiclassical treatment of the TPRM is presented in section 2 and Section 3 highlights the results and discussion.

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2. Semiclassical Hamiltonian for the TPRM

The Hamiltonian for the TPRM with single particle in high-j $(= h_{\frac{11}{2}})$ configuration is given by

$$\begin{aligned} H &= \sum_k \frac{{R_k}^2}{2\mathcal{J}_k} + \kappa r^2 \beta \Big[\cos\gamma \langle Y_{20}\rangle \\ &\quad + \frac{1}{\sqrt{2}} \sin\gamma \langle Y_{22} + Y_{2-2}\rangle\Big] \\ &= \sum_k \Big[A_k (I_k - j_k)^2 + Q_k {j_k}^2\Big]. \qquad (1) \end{aligned}$$

Here, Q_k $(k = 1-3)$, the mean field parameters depend on the sub-shell filling where $q_0 (= 206 A^{-\frac{1}{3}} \beta \sqrt{\frac{5}{4\pi}} \frac{3}{4j(j+1)})$ and an orientation degree of freedom γ $(0 \le \gamma \le 60^0$ [8]), and are defined as

$$Q_1 = -Q_2 = \frac{q_0}{\sqrt{3}} \sin\gamma; \qquad Q_3 = q_0 \cos\gamma. \qquad (2)$$

Here A_k's $((k = 1-3))$ are the rotational parameters. If we use $\gamma = 0^0$, the standard axially symmetric PRM Hamiltonian is recovered.
The equations of motion for the components of angular momentum I_k and j_k are obtained from the Hamiltonian (1) by using the Poisson bracket algebra of angular momenta

$$\frac{\partial I_k}{\partial t} = -2\epsilon_{klm} A_l (I_l - j_l) I_m, \qquad (3)$$

$$\frac{\partial j_k}{\partial t} = 2\epsilon_{klm} \Big[-A_l (I_l - j_l) + Q_l j_l\Big] j_m. \qquad (4)$$

Here, ϵ_{klm} is the unit antisymmetric tensor. For fixed values of I $(=\sqrt{{I_1}^2 + {I_2}^2 + {I_3}^2})$ and j $(=\sqrt{{j_1}^2 + {j_2}^2 + {j_3}^2})$, two different types of stationary states arise: (i) the axes aligned and (ii) the planar. The plane confined stationary states have components of $\vec{I}$ and $\vec{j}$ in one of the k-l plane and involve γ degree of freedom. Due to triaxiality, the resultant angular momentum lies out of the plane and hence generates a pair of degenerate $\Delta I = 1$ bands.

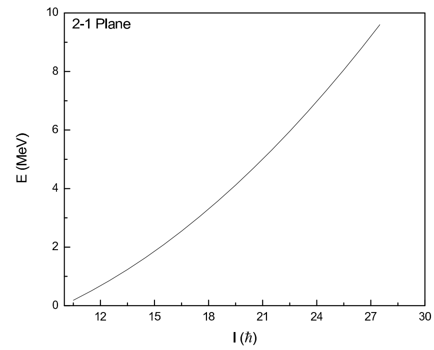

FIG. 1: Plot of excitation energy, E vs the angular momentum I, for the 2-1 planar stationary states.

3. Results and Discussion

We have tested our model for an odd-A nucleus ^{135}Nd, in which the chiral pair has been seen[9]. In this nucleus, the 75th neutron occupies $h_{\frac{11}{2}}$ shell and therefore, fixes the single particle angular momentum, $j = 5.5\hbar$. A self-consistent minimization of the total energy fixes $\beta(= 0.196)$ and $\gamma(= +28^o)$. Our calculations fix the stationary state vectors $\vec{I}$ and $\vec{j}$ in the 2-1 plane. Figure 1 shows the plot of excitation energy, E vs the angular momentum I, for the 2-1 planar stationary states. It is obvious from this plot that the bandhead appears at an angular momentum, I=10.5$\hbar$, which is fairly close to an experimental value (I=11.5$\hbar$). Also, this plot reveals that the variation of angular momentum, I, lies within the limits $10.5\hbar \le I \le 27.5\hbar$. This implies that a fairly large band is supported on such a triaxial shape. The triaxial parameter, $\gamma(= +28^o)$ implies that the moment of inertia is very large along principal axes 1 and restricts the precession to take place along this axis. Small oscillations developed around the planar fixed points break down the PT symmetry and manifests two spectra, namely, Semi-1 and Semi-2. These spectra are shown in figure 2. In this figure, Tilted axes cranking (TAC) results are also shown. The observed negative parity $\Delta I = 1$ chiral bands, namely A and B of ^{135}Nd nucleus are plotted for comparison. It is remarkable to note that semiclassical results are fairly in agreement with the observed spectra of ^{135}Nd, whereas the TAC results are unable to differentiate two bands.

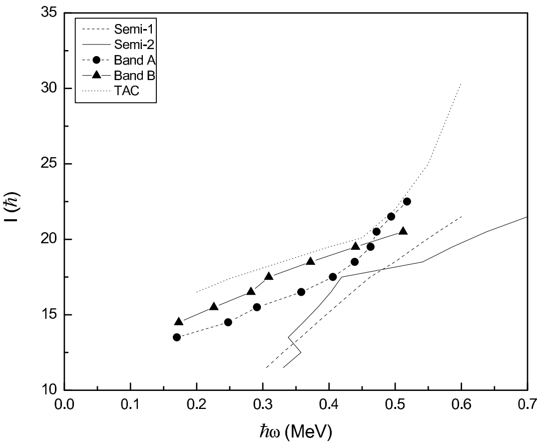

FIG. 2: Plot of semiclassical and the TAC results of angular momentum I vs the rotational frequency. The experimental results for the negative parity $\Delta I = 1$ chiral bands, namely A and B of ^{135}Nd nucleus are also shown

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