Quasiparticle spectra via Kolmogorov stochasticity parameter

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Introduction

Spectral fluctuations in nuclear spectra have been studied in a great detail for last sixty years. The central aim is to classify various nuclei in terms of underlying symmetries, and with a semiclassical view, also in terms of underlying nucleon dynamics. The symmetry classification was made clear by Wigner and Dyson and this is well-explained in the classic text by Mehta [1]. One can also see numerous instances when spectral fluctuations are found to be in an excellent agreement with various universality classes of random matrices. The rationale for why Random Matrix Theory (RMT) works well in not completely clear as yet. One of the requirements of RMT to work is that the level sequences should be large enough. However, often in nuclear physics, we have only a limited number of energy levels of a given system. For instance, in the study presented here, there are only fifty two levels. When we employ RMT for such a case, we can never be confident about the results. Using RMT, when we obtain the nearest-neighbour level spacing distribution, we get a Poisson distribution. This indicates a regularity in the underlying dynamics at the classical level of description. However, with the manifestly obvious presence of many-body chaos, it becomes important to examine the analysis.

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Stochasticity parameter

Define stochasticity parameter for a sequence of length n,

$$\lambda_n = \sup_E \frac{|N_n(E) - N_0(E)|}{\sqrt{n}} \qquad (1)$$

where $N_n(E)$ is the cumulative density of energy levels and $N_0(E)$ is the average cumulative density. In the limit of large n, the difference $N_n(E) - N_0(E)$ will be just the oscillating part of the density of levels. The parameter λ_n itself is a random variable and we need to study its distribution function. We obtain the cumulative probability distribution function $\Phi(\Lambda)$ of variable, λ which gives the probability of the stochasticity parameter to have a value $\lambda \leq \Lambda$. The stochastic probability of a system with Kolmogorov parameter, Λ_0 is $\Phi(\Lambda_0)$.

Results

We have shown [2] that integrable systems follow a density distribution (prime denoting the derivative with respect to the argument)$\Phi'(\Lambda) \sim \exp(-\Lambda^\beta)$. For a typical integrable system, β is around 3 (e.g. 2.93 for a circular billiard). For a chaotic system, $\Phi'(\Lambda) \sim (1 + \Lambda^{-\gamma})$. For standard map, it is about 3.6 whereas for Riemann zeros, it is about 4.4. For independent identically distributd random variables, the distribution has the well-known Kolmogorov form [2].

In this study on three-quasiparticle states, we present results of statistical analysis of the one-quasiparticle (1qp) and three-quasiparticle (3qp) excitations by calculating a stochasticity parameter introduced by Kolmogorov. Our objective for this statistical study is to test whether there is any change in the fluctuation properties (i.e. any transition from chaotic to integrable domain or vice versa) while going from the 1qp excitations to the 3qp excitations. While discussing 3qp states, there are two issues - (i) collectivity, (ii) three-body effect.

We examine the behavior of the nuclear energy levels, which corresponds to the 1qp excitations and the 3qp excitations in deformed nuclei. At excitation energy $\sim$ 1MeV, which is approximately the energy gap 2Δin the rare-earth region, a proton or a neutron pair can breakup and form a 3qp state in an odd-A nucleus. Such breakings of a proton/ neutron pair may lead to possible symmetry breakings and appearance of some three-body effects. In order to test such effects while going from the 1qp excitations to the 3qp excitations, we did a comparison of the fluctuation properties of the nuclear energy levels, which correspond to the 1qp excitations and the 3qp excitations.

For our calculations, we have used 52 1qp and 49 3qp levels having the spin and parity of $\frac{27}{2}^+$. Choosing the same spin-parity ensures that the levels have the same symmetries. We have taken the data for the 1qp states from the Evaluated Nuclear Structure Data File (ENSDF) [3] and for the 3qp states from our compilation [4]. The mass region selected for the 1qp and the 3qp excitations is $153 \leq A \leq 181$ with $63 \leq Z \leq 76$ for the 1qp states and $65 \leq Z \leq 77$ for the 3qp states.

The calculations show that the 1qp and 3qp levels follow the density distribution function,$\Phi'(\Lambda) \sim a\exp(-x^b)$, $\Phi'(\Lambda) \sim (c + \Lambda^{-\gamma})$ respectively as expected. The PDF and the fitted forms are shown in Fig.1,2. On the contrary, stochastic probability is about 0.43 and 0.99 for 1qp and 3qp levels. The results clearly show that the existence of underlying chaoticity or integrability can be made on the basis of PDF even for a modest size of 50 data points, but to calculate exact stochastic probability the larger data set will be required to overcome the statistical errors (as the limiting distributions are still numerical). Since both qp states are of many-body nature, the origin of chaos as manifested in the spectrum is most probably due to an effective three-body interaction among three quasi-particles.

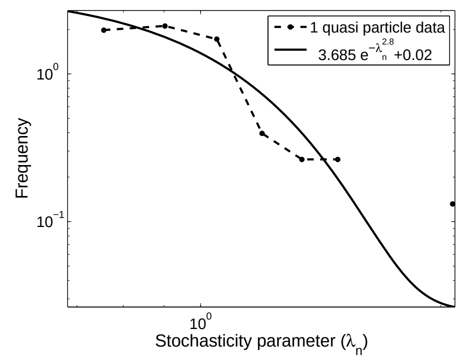

FIG. 1: PDF for 1 quasi-particle levels.

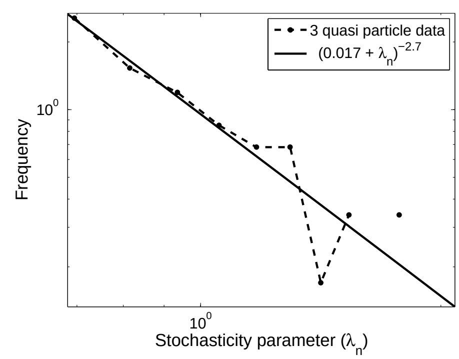

FIG. 2: PDF for 3 quasi-particle levels.

References

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