Structure of ΛΛ hypernuclei with the Skyrme-Hartree-Fock Theory

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Hypernuclear spectroscopy provides an invaluable tool to get information on the many-body hadronic systems utilizing the strangeness degrees of freedom. The investigation of the structure of hypernuclei is important because it provides the opportunity to learn about the bare hyperon-nucleon and also the hyperon-hyperon interactions which are needed for the investigation of the new aspects and new forms of the hadronic matter. In view of the recent new developments of the hypernuclear physics (see, e.g. [1]), there is quite some interest in developing a self-consistent description of the hypernuclei.

In this paper we have used extended Skyrme-Hartree-Fock theory to calculate the improved interaction strength parameters of ΛN effective interaction. The parameters sets are obtained so as to reprodue the Λ single particle energies of single Λ hypernuclei, binding energy and radius of known double Λ hypernuclei over the whole periodic table. The total energy density functional (EDF) of Λ hypernucleus with in the extended density dependent Skyrme Hartree Fock theory is expressed as,

$$\mathcal{E}^H_{1\Lambda} = \mathcal{E}_{NN}(\rho_n, \rho_p, \tau_n, \tau_p, J_n, J_p) + \mathcal{E}_{\Lambda N}(\rho_n, \rho_p, \rho_\Lambda, \tau_\Lambda) + \mathcal{E}_{Pair}(v_p, v_n) + \mathcal{E}^\Lambda_R(\rho_n, \rho_p, \rho_\Lambda). \quad (1)$$

Where, $\mathcal{E}_{NN}$ is the original Skyrme Hartree-Fock nuclear Hamiltonian density based upon the nucleon-nucleon interactions. We employ SLy4 [2] Skyrme parameterization to calculation the energy density functional of core nucleus ${}^{A-1}_{\Lambda}Z$ of hypernucleus. The ρ_i, (i = n,p, and Λ) one body density, τ_i kinetic density, $J_{n/p}$ the spin orbit current operator, and $v_{n/p}$ are the occupation probabilities of nucleons in the core nucleus. $\mathcal{E}_{\Lambda N}$ is ΛN interaction term, $\mathcal{E}_{Pair}(v_p, v_n)$ is the pairing density functional, and $\mathcal{E}^\Lambda_R(\rho_n, \rho_p, \rho_\Lambda)$ is contribution from rearrangement energy functional. The total energy density functional for ΛΛ hypernuclei in written as,

$$\mathcal{E}^H_{2\Lambda} = \mathcal{E}^H_{1\Lambda} + \mathcal{E}^H_{\Lambda\Lambda}(\rho_N, \rho_\Lambda) \quad (2)$$

The double Λ energy density may written as,

$$\mathcal{E}^H_{\Lambda\Lambda}(\rho_N, \rho_\Lambda, \tau_\Lambda) = \int d^3r H_{\Lambda\Lambda}(r), \quad (3)$$

the Hamiltonian density $H_{\Lambda\Lambda}$ is written by using two-body and three body Skyrme interaction forces,

$$H_{\Lambda\Lambda} = \frac{1}{4}\lambda_0\rho^2_\Lambda + \frac{1}{8}(\lambda_1 + 3\lambda_2)\rho_\Lambda\tau_\Lambda + \frac{3}{32}(\lambda_2 - \lambda_1)\rho_\Lambda\nabla^2\rho_\Lambda + \frac{1}{4}\lambda_3\rho^2_\Lambda\rho^\gamma_N. \quad (4)$$

The double Λ hyperon interaction parameters, $\lambda_0 - \lambda_3$ and γ of the energy functional are obtained from fitting to experimental data of double Λ binding energy $B_{\Lambda\Lambda}$ and ΛΛ interaction energy $\Delta B_{\Lambda\Lambda}$ of the hypernuclei.

In Table I, we present the different sets of parameterizations for ΛN effective interaction obtained by the methods of successive approximation for χ^2 minimization. In case of ΛN effective interaction parameters, we calculated data set by fitting experimental data set consists of Λ single particle energies of hypernuclei, ${}^8_\Lambda$He, ${}^9_\Lambda$Li, ${}^{10}_\Lambda$Be, ${}^{10}_\Lambda$B, ${}^{11}_\Lambda$B, ${}^{12}_\Lambda$B, ${}^{12}_\Lambda$C, ${}^{13}_\Lambda$C, ${}^{16}_\Lambda$O, ${}^{17}_\Lambda$O, ${}^{28}_\Lambda$Si, ${}^{32}_\Lambda$S, ${}^{33}_\Lambda$S, ${}^{40}_\Lambda$Ca, ${}^{41}_\Lambda$Ca, ${}^{51}_\Lambda$V, ${}^{56}_\Lambda$Fe, ${}^{89}_\Lambda$Y, ${}^{139}_\Lambda$La, and ${}^{208}_\Lambda$Pb, with Λ particle in 1s, 1p and 1d orbitals. We get values of ΛN potential, $V_{\Lambda N}$ = 28.00±0.58 MeV at nuclear matter density ρ= 0.16±0.01fm^{-1} for the data of hypernuclei employed in the χ^2 minimization. The u_0 and y_0, are the parameters of zero

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TABLE I: A new parameterizations for Λ-N Skyrme potential derived self-consistently by fitting a large set of experimental data of Λ single particle energies for two different values of β.

SET	β	u_0 (MeV fm^3)	u_1 (MeV fm^5)	u_2 (MeV fm^5)	u_3 (MeV fm$^{3+3\beta}$)	y_0	y_3	χ^2
HP060310	1	-329.1081	39.5589	19.9098	1429.8016	-0.1703	0.3160	1.4651
HP080310	1	-523.7681	63.2191	54.6234	1884.7219	-0.7703	-0.3495	1.3478
HP100310	1	-418.4806	49.3298	40.4601	2050.3923	-0.5138	-0.4580	1.3847
HP120310	1	-346.5556	98.8998	50.3842	1617.5662	-0.1352	-0.3892	1.3177
HP160310	1/3	-483.3032	96.3052	64.2609	1114.5066	-0.2446	-0.4741	1.4095
HP180310	1/3	-450.5685	148.0403	-20.7068	1491.2256	-0.0104	-0.5861	1.4719
HP230310	1/3	-506.6520	129.4331	19.0171	1368.7793	-0.2597	-0.5879	1.4369
HP270310	1/3	-504.2675	133.9351	-2.9475	1811.9971	-0.2584	-0.8989	1.4468

TABLE II: The theoretical results of ΛΛ hyperon in various hypernuclei calculated using Skyrme parameters are compared with available experimental data. The ΛΛ particles occupies $s_{1/2}$ state and $r_{\Lambda\Lambda}$ is the rms radius of ΛΛ orbitals. The theoretical results are computed by using Sly4 force for nucleon-nucleon interaction, set HP060310 parametrization is employed to calculate the ΛN interaction and different set of parametrization are used for calculating ΛΛ interactions [3].

SET	Hypernuclei	$B_{\Lambda\Lambda}$	$B_{\Lambda\Lambda}$(Expt.)	BE/Baryon	$r_{\Lambda\Lambda}$	$V_{\Lambda\Lambda}$	ρ_0	χ^2
*SLL*1	$^{6}_{\Lambda\Lambda}$He	3.75	2.8 ± 0.8	4.05	3.71	-2.59	0.15	1.28
	$^{7}_{\Lambda\Lambda}$Li	4.70	5.7 ± 0.8	3.41	3.78	-2.47	0.15	
	$^{9}_{\Lambda\Lambda}$Be	6.96	8.1 ± 0.8	5.38	3.34	-2.45	0.16	
	$^{10}_{\Lambda\Lambda}$Be	8.08	8.4 ± 0.8	6.38	3.21	-2.43	0.16	
S*LL*2	$^{6}_{\Lambda\Lambda}$He	3.68	2.8 ± 0.8	4.04	3.80	-2.85	0.15	1.37
	$^{7}_{\Lambda\Lambda}$Li	4.63	5.7 ± 0.8	3.40	3.89	-2.75	0.15	
	$^{9}_{\Lambda\Lambda}$Be	6.89	8.1 ± 0.8	5.37	3.41	-2.76	0.16	
	$^{10}_{\Lambda\Lambda}$Be	8.00	8.4 ± 0.8	6.37	3.26	-2.75	0.16	
S*LL*3	$^{6}_{\Lambda\Lambda}$He	3.39	2.8 ± 0.8	3.99	4.08	-3.51	0.15	1.99
	$^{7}_{\Lambda\Lambda}$Li	4.32	5.7 ± 0.8	3.36	4.23	-3.45	0.15	
	$^{9}_{\Lambda\Lambda}$Be	6.56	8.1 ± 0.8	5.33	3.64	-3.53	0.16	
	$^{10}_{\Lambda\Lambda}$Be	7.67	8.4 ± 0.8	6.33	3.46	-3.54	0.16	
S*LL*4	$^{6}_{\Lambda\Lambda}$He	3.54	2.8 ± 0.8	4.01	3.95	-3.02	0.15	1.63
	$^{7}_{\Lambda\Lambda}$Li	4.48	5.7 ± 0.8	3.38	4.07	-2.95	0.15	
	$^{9}_{\Lambda\Lambda}$Be	6.73	8.1 ± 0.8	5.35	3.53	-2.99	0.16	
	$^{10}_{\Lambda\Lambda}$Be	7.85	8.4 ± 0.8	6.35	3.36	-3.00	0.16	

range term, u_1 and u_2 are representing the effective and finite range term, and u_3 and y_3 are the parameters of density dependent term of the Hamiltonian density of hypernucleus.

Table II provide us the theoretical results of ΛΛ hyperon in various hypernuclei. It shows the comparison of theoretical calculated binding energy and respective known experimental binding energies of various double Λ hypernuclei. There is good agreement between the two as χ^2 obtained in each case is less than 2.0. The double Λ potential obtained is $V_{\Lambda\Lambda}$=(3-4)MeV at nuclear matter saturation density ρ_0=0.16fm^{-3} and also radius $r_{\Lambda\Lambda}$=$(3-4)$ fm.

References

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