Energy of the excited degenerate doublet ($3^+/2$, $5^+/2$) of ${}^{13}_{\Lambda}$C

Mohammad Shoeb* and Sonika
Department of Physics, Aligarh Muslim University, Aligarh-202002, INDIA

Introduction

Alpha cluster model [1–7] has been successfully used in explaining the ground and excited states of p-shell hypernuclei. In our previous work [6] we have calculated the energies of the ground and 2^+ states of ^{12}C in α-cluster model. In Table I we have given experimental values and listed the ground and excited state energies from earlier work [3–7].

In the recent past, Shoeb and Sonika [3] have analysed the energy of the ground state of ${}^{13}_{\Lambda}$C in the α cluster model using $\alpha\alpha\alpha$ [3] and phenomenological dispersive [1] three-body $\Lambda\alpha\alpha$ forces along with two-body cluster forces.

The attractive three-body $\alpha\alpha\alpha$ Gaussian shape potential has the form:

$$V_{\alpha\alpha\alpha} = W_3 \exp\big(- (r_{23}^2 + r_{24}^2 + r_{34}^2)/\alpha^2\big), \quad (1)$$

with the strength $W_3(= -16.0$ MeV) and the range parameter $\alpha(= 7.7$ fm).

$V_{\Lambda\alpha\alpha}$, the phenomenological dispersive three-body $\Lambda\alpha\alpha$ potential, has a simple form

$$V_{\Lambda\alpha\alpha} = W_0 f(r_{\Lambda\alpha_1}) f(r_{\Lambda\alpha_2}), \quad (2)$$

where the strength $W_0 > 0$ gives repulsion. The radial factor $f(r)$ is of Yukawa form: $f(r) = \exp(-ar)/ar$, where a is the range parameter. The $V_{\Lambda\alpha\alpha}$ ($W_0 = 17.0$ MeV and $a = 0.5$ fm^{-1} for the case of two-body $\Lambda\alpha$ Isle potential) is constrained to fit the B_Λ, Λ-binding energy [1] of the ${}^{9}_{\Lambda}$Be in the three-body $\Lambda\alpha\alpha$ model. The inclusion of the dispersive $\Lambda\alpha\alpha$ force [1, 2] gives a good account of the ground and excited states of ${}^{9}_{\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be. Further, two-body $\alpha\alpha$ potential of Ali and Bodmer [4] and $\Lambda\alpha$ potential Isle type [1] were constrained by the experimental data on the two-body cluster. The calculated ground state energy of ${}^{13}_{\Lambda}$C is given in Table II. The satisfactory explanation [3] of the ground state energy of ${}^{13}_{\Lambda}$C in the $\Lambda\alpha\alpha\alpha$ model using variational Monte Carlo (VMC) method has motivated us to apply α-cluster model to predict the energy of the excited degenerate doublet ($3^+/2$, $5^+/2$) of ${}^{13}_{\Lambda}$C.

Hamiltonian, wavefunction and energy calculation

The excited ($3^+/2$, $5^+/2$) state of ${}^{13}_{\Lambda}$C is a coupled state of s_Λ=1/2 and 2^+ of ^{12}C. The Hamiltonian in the $\Lambda\alpha\alpha\alpha$ model ignoring small Λ spin-orbit force has the form:

$$\begin{aligned} H_\Lambda \;=\; & K_\Lambda(1) + \sum_{i=2}^{4} K_\alpha(i) + \sum_{i=2}^{4} V_{\Lambda\alpha}(r_{1i}) \\ & + \sum_{i<j} \sum_{i=2,3,j=3,4} V^l_{\alpha\alpha}(r_{ij}) \\ & + \sum_{i<j} \sum_{i=2,3,j=3,4} V_{\Lambda\alpha\alpha}(r_{1i}, r_{1j}) \\ & + V_{\alpha\alpha\alpha}(r_{23}, r_{24}, r_{34}), \end{aligned} \quad (3)$$

where indices (1), (2, 3, 4) label Λ, and αs particles, respectively, K_Y is the kinetic energy operator for the particle $Y(=\Lambda, \alpha)$, V_{hh} denotes the potential for the pair of particles $hh(= \Lambda\alpha, \alpha\alpha)$, r_{ij} is the inter-particle separation for the pair having indices i and j. The two-body $V^l_{\alpha\alpha}$ potential in the relative angular momentum d-state for the $\alpha\alpha$ pair (34) and in s-state for the remaining pairs have been given in our earlier paper [6].

The trial wavefunction for the excited ${}^{13}_{\Lambda}$C is the product of two-body correlation functions f_{hh} for the pair of particles, hh and the appropriately coupled spin and angular function

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*Electronic address: `mshoeb202002@gmail.com`

TABLE I: The calculated energies of ground and excited 2^+ states of ^{12}C are listed in column four for the states given in third column. Energies in bold face are from analyzes [4, 5, 7] (Experimental energy ^{12}C (g.s.) $E_B = -7.26$ MeV, ^{12}C (2^+) = -2.84 MeV.

	System	States	-E_B (MeV)
Our work Ref. [3]	^{12}C	0^+	7.17
Ref. [4]			**6.81**
Ref. [5]			**7.26**
Ref. [6]		2^+	4.29
Ref. [7]			**4.99**

TABLE II: The calculated energies of ground and excited degenerate ($3^+/2$, $5^+/2$) states of $^{13}_{\Lambda}$C are listed in column four for the states given in third column. Energy in bold face is from analysis [7].

	System	States	-E_B (MeV)
Our work Ref. [3]	$^{13}_{\Lambda}$C	0^+	18.81
Present work		($3^+/2$, $5^+/2$)	14.98
Ref. [7]			**17.00**

($\zeta = s_\Lambda \otimes y_{2m}(\Omega_{34})$) and has the form:

$$\Psi_\Lambda = \left[\prod_{i=2}^{4} f_{\Lambda\alpha}(r_{1i})\right] \times \left[\prod_{i<j_{i=2,3,j=3,4}} f^{l}_{\alpha\alpha}(r_{ij})\right]\zeta. \tag{4}$$

The correlation functions $f_{hh}(r)$ are obtained from a procedure developed by Urbana group.

The total energy E_B for the system $^{13}_{\Lambda}$C in the cluster model for the trial wavefunction Eq. (4) is evaluated using the following relation:

$$E_B(^{13}_{\Lambda}C) = \frac{\langle\Psi_\Lambda|H_\Lambda|\Psi_\Lambda\rangle}{\langle\Psi_\Lambda|\Psi_\Lambda\rangle}. \tag{5}$$

The VMC estimates of the energy were made for 100 000 points and the energy was optimized with respect to variational parameters using standard computer code.

Results and Discussion

The predicted energy (Table II) of excited ($3^+/2$, $5^+/2$) state of $^{13}_{\Lambda}$C in the VMC framework is -14.98 MeV which is higher by 2.0 MeV than the one predicted by Hiyama *et. al.* [7]. In the absence of experimentally measured energy, it is not possible to comment on whether our VMC or Correlated Gaussian basis function method [7] is appropriate for the excited state of $^{13}_{\Lambda}$C.

The root mean square (RMS) radii for various $\alpha\alpha$ pairs: $\mathbf{R}_{\alpha_2\alpha_3} = 3.82$ fm, $\mathbf{R}_{\alpha_2\alpha_4} = 3.82$ fm, $\mathbf{R}_{\alpha_3\alpha_4} = 3.43$ fm. RMS distance between center of mass (CM) of 3-alpha and Λ, $\mathbf{R}_{(3\alpha)\Lambda} = 2.44$ fm; between CM of 3-alpha and αs, $\mathbf{R}_{(3\alpha)\alpha_2} = 2.28$ fm, $\mathbf{R}_{(3\alpha)\alpha_3} = 2.06$ fm, and $\mathbf{R}_{(3\alpha)\alpha_4} = 2.06$ fm.

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