Bounds on new Polarization parameters of an entangled channel spin-1 system

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Introduction

Entanglement, which is considered to be the most nonclassical manifestation of quantum formalism is a valuable resource for quantum computation and quantum information. Here we study entanglement of a channel spin-1 system which can be realised using polarised spin 1/2 beam and polarised spin 1/2 target which naturally arise in nuclear physics experiments like hadron scattering and reaction processes. This discussion is best done by employing a new representation for the density matrix which provides physical interpretation for the polarization parameters. These parameters are real and shown to be related to the expectation values, variances and co-variances of spin operators J_x, J_y and J_z. Bounds on these parameters for entangled regions of channel spin-1 system are discussed.

A new representation of spin-1 density matrix

The standard expression for the density matrix 'ρ' for a spin j system in terms of irreducible tensor operators τ^k_q is given by

$$\rho = \frac{Tr(\rho)}{(2j+1)} \sum_{k=0}^{2j} \sum_{q=-k}^{+k} t^k_q \, \tau^{k^\dagger}_q \ , \qquad (1)$$

where

$$t^k_q = \frac{Tr(\rho\,\tau^k_q)}{Tr\rho}.$$

Since ρ is Hermitian and $\tau^{k^\dagger}_q = (-1)^q\,\tau^k_{-q}$, t^k_q satisfy the condition

$$t^{k^*}_q = (-1)^q\, t^k_{-q}.$$

Following the well known Weyl construction [1],

$$\tau^k_q(\vec{J}) = \mathcal{N}_{kj}\,(\vec{J}\cdot\vec{\bigtriangledown})^k\, r^k\, Y^k_q(\hat{r})\,, \qquad (2)$$

where $\mathcal{N}_{kj}$ is the normalization factor and $Y^k_q(\hat{r})$ are the spherical harmonics.

Considering the particular case of spin-1 density matrix, we now define a complete set of Hermitian, linearly independent operators M^0, M^1, ..., M^8 as follows.

$$M^0 = \sqrt{\frac{2}{3}}\tau^0_0\ ,\quad M^1 = \frac{\tau^1_1 + \tau^{1\dagger}_1}{\sqrt{3}}\ ,$$

$$M^2 = \frac{i(\tau^1_1 - \tau^{1\dagger}_1)}{\sqrt{3}}\ ,\quad M^3 = \sqrt{\frac{2}{3}}\tau^1_0,$$

$$M^4 = \frac{i(\tau^2_2 - \tau^{2\dagger}_2)}{\sqrt{3}}\ ,\quad M^5 = \frac{i(\tau^2_1 - \tau^{2\dagger}_1)}{\sqrt{3}},$$

$$M^6 = \frac{\tau^2_1 + \tau^{2\dagger}_1}{\sqrt{3}},\ M^7 = \frac{\tau^2_2 + \tau^{2\dagger}_2}{\sqrt{3}},\ M^8 = \sqrt{\frac{2}{3}}\tau^2_0.$$

and the corresponding matrices in angular momentum basis are

$$M^0 = \sqrt{\frac{2}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\ ,\ M^1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0&-1&0\\-1&0&-1\\0&-1&0\end{pmatrix},$$

$$M^2 = \frac{i}{\sqrt{2}}\begin{pmatrix}0&-1&0\\1&0&-1\\0&1&0\end{pmatrix}\ ,\ M^3 = \begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix},$$

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$$M^4 = i\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad M^5 = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix},$$

$$M^6 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad M^7 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$M^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Observe that the above matrices are normalized i.e., $Tr(M^k M^{k'}) = 2\,\delta_{kk'}$ and $M^1, ..., M^7$ have eigen values -1, 0, +1. In this representation the density matrix for the most general spin-1 system can be written as

$$\rho^1 = \frac{1}{2}\sum_{i=0}^{8} r^i M^i, \qquad (3)$$

where $r^i = Tr(\rho M^i)$. Explicitly,

$$r^1 = \langle J_x\rangle\ ,\quad r^2 = \langle J_y\rangle\ ,\quad r^3 = \langle J_z\rangle,$$

$$r^4 = -\langle J_xJ_y + J_yJ_x\rangle\ ,\quad r^5 = \langle J_yJ_z + J_zJ_y\rangle\ ,$$

$$r^6 = -\langle J_xJ_z + J_zJ_x\rangle\ ,\quad r^7 = \langle J_x^2\rangle - \langle J_y^2\rangle\ ,$$

$$r^8 = 3\langle J_z^2\rangle - 2.$$

As r^i's are constructed out of the moments of $\hat{J}_x$, $\hat{J}_y$ and $\hat{J}_z$, they constitute an experimentally measurable set of parameters.

Let us now consider the example of channel spin-1 system. If both the spin-1/2 beam and the spin-1/2 target are prepared to be in mixed states, then the corresponding density matrices are given by

$$\rho(i) = \frac{1}{2}\,[I + \vec{\sigma}(i)\cdot\vec{p}(i)], \quad i = 1, 2. \qquad (4)$$

Here $\vec{p}(i)$ are the polarization vectors and $\vec{\sigma}(i)$'s are the Pauli spin matrices.

In special Lakin frame (SLF) which is widely used in studying nuclear reactions, if $|\vec{p}(1)| = |\vec{p}(2)| = p$, the density matrix for channel spin-1 system assumes the form [2]

$$\rho = \frac{1}{N}\begin{pmatrix} (1+pcos\theta)^2 & 0 & -p^2sin^2\theta \\ 0 & 1-p^2 & 0 \\ -p^2sin^2\theta & 0 & (1-pcos\theta)^2 \end{pmatrix} \qquad (5)$$

where N $= (3 + p^2cos2\theta)$, 2θ is the angle between polarization vectors.

Comparing the above matrix with the new representation as given in equation (3), the expressions for non-zero parameters r^3, r^7 and r^8 are

$$r^3 = \frac{4pcos\theta}{3 + p^2cos2\theta}\ ,\quad r^7 = -\frac{2p^2sin^2\theta}{3 + p^2cos2\theta}\ ,$$

$$r^8 = \frac{2p^2(1 + cos^2\theta)}{\sqrt{3}\,(3 + p^2cos2\theta)}.$$

Since $\rho \geq 0$ (positive semidefinite), the bounds on r^i's are found to be

$$-\frac{2}{\sqrt{3}} \leq r^8 \leq \frac{1}{\sqrt{3}}\ ,\quad (r^7)^2 + (r^3)^2 \leq (\frac{2}{3} + \frac{r^8}{\sqrt{3}})^2\ ,$$

$$-(\frac{2}{3} + \frac{r^8}{\sqrt{3}}) \leq r^3 \leq (\frac{2}{3} + \frac{r^8}{\sqrt{3}})\ .$$

Using the well known co-variance matrix formalism , a channel spin-1 system in SLF is entangled iff the eigen correlation $C_{xx} = \frac{1}{3} - \frac{r^8}{\sqrt{3}} + r^7 < 0$ [2]. Since $r^8 \geq 0$ and $r^7 \leq 0$ for this system, in the limit $r^8 = 0$, the system is entangled iff $|r^7| > \frac{1}{3}$. When $r^8 = \frac{1}{\sqrt{3}}$, the system is entangled.

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References

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