Neutron induced reaction cross sections for ADSS

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Introduction

We derive an analytical model of simple functional form from nuclear reaction theory and demonstrate the quantitative agreement with the measured cross sections for neutron induced reactions. The neutron-nucleus total, scattering and reaction cross sections, for energies upto 700 MeV and for several nuclei spanning a wide mass range are estimated. Systematics of neutron scattering cross sections on various materials for neutron energies upto several hundred MeV are important for ADSS applications. The reaction cross sections of neutrons are useful for determining the neutron induced fission yields in actinides and pre-actinides. The present model provides very good estimates of the total cross section for neutron induced reaction.

Theoretical Formalism

The basic picture of the nuclear Ramsauer model is that the forward scattering amplitude for a neutron incident on a nucleus is given by

$$f(0^o) = \frac{i}{2k}\sum_{l=0}^{L}(2l+1)(1-e^{2i\delta_l}) \qquad (1)$$

where the complex phase shift δ_l may be considered as independent of angular momenta l of the lth partial wave, so that the above expression, after summing over l upto a maximum value $L = kR_{ch}$, reduces to

$$f(0^o) = ik(R_{ch}+\lambda)^2(1-\alpha e^{i\beta})/2 \qquad (2)$$

where R_{ch} is the channel radius beyond which partial waves do not contribute. The quantity $e^{2i\delta_l}$ is replaced by $\alpha e^{i\beta}$ with β being two times the real part of the phase shift and α accounting for the attenuation or loss of flux (arising out of imaginary part of the phase shift). In this approach since the total cross section is related to the imaginary part of the forward scattering amplitude by

$$\sigma_{tot} = \frac{4\pi}{k} Im f(0^o), \qquad (3)$$

the total cross section (σ_{tot}), the scattering cross section (σ_{sc}) and the reaction cross section (σ_r) are given by

$$\begin{aligned}\sigma_{tot} &= 2\pi(R_{ch}+\lambda)^2(1-\alpha\cos\beta),\\ \sigma_{sc} &= \pi(R_{ch}+\lambda)^2(1+\alpha^2-2\alpha\cos\beta),\\ \sigma_r &= \pi(R_{ch}+\lambda)^2(1-\alpha^2).\end{aligned} \qquad (4)$$

where $\lambda = 1/k = \hbar/\sqrt{2mE}$, the incident neutron energy in the center of mass system is E and m is the neutron mass.

The optical potential for nuclear n-N interaction can be written as $-V - iW$ with V and W as positive quantities, and contains no Coulomb interaction. The phase shift δ in a WKB approximation is $[\int Kdr - \int kdr]$ and real part of it to a zero order approximation for a square well with radius R is $(K-k)R$ and hence β is determined by the real potential V,

$$\beta = 2[K-k]R = \frac{\sqrt{8m}}{\hbar}[\sqrt{E+V}-\sqrt{E}]R \quad (5)$$

while the attenuation factor α is determined primarily by the imaginary potential W,

$$\alpha = e^{-\bar{R}/\Lambda} = e^{-2mW\bar{R}/\hbar^2K} \qquad (6)$$

where Λ is the mean free path of the neutron inside the nucleus, $K = \sqrt{2m(E+V)}/\hbar$ and $k = \sqrt{2mE}/\hbar$ are the real wave numbers inside and outside the nucleus respectively. The average chord length $\bar{R}$ of a neutron passing through a nucleus can be derived as

$$\bar{R} = \frac{\int_0^R 2\sqrt{R^2-x^2}(I2\pi x dx)}{\int_0^R I2\pi x dx} = \frac{4}{3}R \qquad (7)$$

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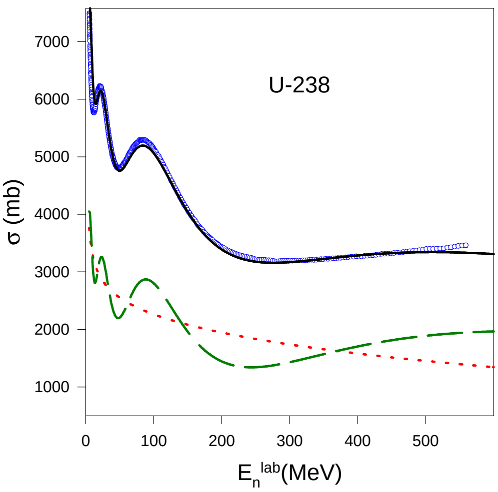

FIG. 1: Cross section versus neutron energy plots for ^{238}U target. Hollow circles and the continuous, dashed and dotted lines represent, respectively, σ_{tot} expt.data [1] and calculated σ_{tot}, σ_{sc} and σ_r.

where I is the neutron flux that is the number of neutrons incident per unit area. As $R \propto A^{\frac{1}{3}}$ ($R \sim r_0 A^{\frac{1}{3}}$), the above arguments imply that

$$\beta = \beta_0 A^{\frac{1}{3}} [\sqrt{E+V} - \sqrt{E}] \quad (8)$$

where $\beta_0 = \frac{r_0\sqrt{8m}}{\hbar}$ whose value is approximately 0.6, and the attenuation factor which is much less than unity but increases with energy (obvious from its expression) is given by

$$\alpha = \exp[-\alpha_0 r_0 A^{\frac{1}{3}} W/\sqrt{E+V}] \quad (9)$$

where $\alpha_0 = \frac{4\sqrt{2m}}{3\hbar}$ whose value turns out to be 0.2929 and r_0 is the nuclear radius parameter. The first term $V_A = V_0 + V_1(1-2Z/A) + V_2/A$ of the real potential $V = V_A + V_E\sqrt{E}$ contains both the isoscalar and the isovector components of the optical potential where Z is the atomic number of the target nucleus, whereas the second term accounts for its energy dependence. The imaginary potential W is taken as $W = W_0 + W_E\sqrt{E+V}$ since the total kinetic energy of the neutron inside the nucleus with attractive potential well of depth V is $E+V$. As the magnitude of the real part of the optical potential decreases with energy while the same for imaginary part increases, this implies that V_E is negative whereas W_E is positive.

Analytical Model Calculations and Results

The present model can be fitted to the experimental neutron total cross sections using Eq.(4). The radius of the nuclear potential is given by $R = r_0 A^{\frac{1}{3}}$ while the channel radius is parametrized as $R_{ch} = r_0 A^{\frac{1}{3}} + r_A\sqrt{E} + r_2$ with $r_A = r_{10}\ \ln A + r_{11}/\ln A$. Ramsauer model fits yield $r_{10} = -0.02298$, $r_{11} = 0.10268$, $r_2 = 0.23216$, $V_0 = 46.51099$, $V_1 = 6.73833$, $V_2 = -117.52082$, $V_E = -3.21817$ and $\beta_0 = 0.5928$. The value of α_0 is kept fixed at 0.2929 and the non linear least square fits yield the value for the imaginary potential $W_0 = 5.29265$ MeV and its energy dependence $W_E = 0.33875$. The nuclear radius parameter r_0 is also fitted reasonably well to 1.37799 $A^{0.00793452}$ fm which means $R = r_0' A^{\frac{1}{3}+\gamma}$ where γ is a very small number compared to $\frac{1}{3}$. These extracted model parameters provide global fits to σ_{tot} spanning a large number of target nuclei.

Summary and Conclusion

We constructed an analytical model, justified it from the optical model and nuclear reaction theory approach and applied it to derive the systematics and performed calculations of neutron-nucleus total, scattering and the reaction cross sections and predicted these cross sections for heavier actinides.The extracted parameters for the present analytical model provide global fits to the neutron total cross sections over a large number of nuclei. We conclude that the present estimates of neutron scattering cross sections are very important for the reactor physics calculations of the ADSS applications and the reaction cross sections are very useful for the theoretical calculations of Radioactive Ion Beam production. These can also be used for performing Hauser-Feshbach calculations to estimate the cross sections for neutron induced fission, evaporation residues or evaporation neutron multiplicities and for comparison of the photon [2] versus neutron induced fission as well.

References

[1] W. P. Abfalterer et al., Phys. Rev. **C 63**, 044608 (2001).

[2] Tapan Mukhopadhyay and D. N. Basu, Phys. Rev. **C 76**, 064610 (2007); Phys. Rev. **C 79**, 017602 (2009).