Coulomb t-matrix effective interaction

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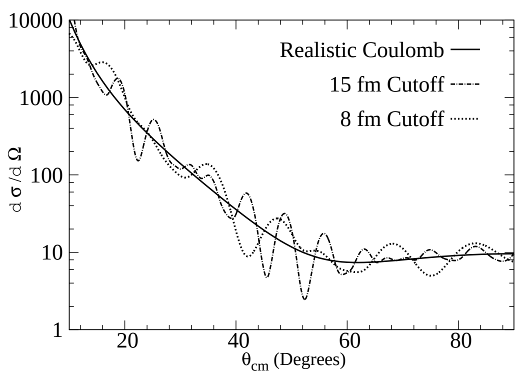
One century ago Rutherford treated the Coulomb scattering classically and obtained the well known Rutherford scattering formula [1]. Quantum mechanical derivation some time later obtained the Coulomb scattering amplitude giving rise to the same Rutherford scattering cross section. Surprising though the classical and quantum mechanical derivations agree remarkably. It is even more surprising that even after the lapse of the century there is no derivation of the t-matrix effective interaction for the Coulomb scattering[2]. The wide range of applicability and utility of Coulomb scattering t_C-matrix effective interaction needs no introduction.

Another remarkable point of the scattering ($\frac{1}{r}$ potential) is the independence of the scattering on the impact parameter and hence the angular momentum. In the conventional evaluation of the t_C-effective interaction one has to incorporate some cut off in the integration (limitation of partial waves as well as range) which leads to highly fluctuating cross section (Seen in Fig. 1 for 105 MeV ^{12}C-^{12}C Rutherford scattering) and hence this procedure is of little or no use.



[t]

FIG. 1: Coulomb scattering using various radil cutoffs in the t-matrix.

We derive the Coulomb t-matrix using the well known quantum mechanical scattering amplitude, $f_c(\theta)$ where θ is the angle of scattering in the center of mass system.

$$f_C(\theta) = \frac{-\eta}{2k\sin^2\frac{\theta}{2}}\exp(-i\eta\ln(\sin^2\frac{\theta}{2}) + 2i\sigma_0) \quad (1)$$

where η is the dimensionless Sommerfield parameter

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar} = \frac{\mu Z_1 Z_2 e^2}{\hbar^2 K} \quad (2)$$

σ_0= Coulomb phase shift for ℓ=0 is given by, σ_0=$\Gamma(1+\imath\gamma)/(\Gamma(1-\imath\gamma)$

Now we have the transition operator T_{fi} related to the scattering amplitude $f(\theta)$ as,

$$T_{fi} = -\frac{2\pi\hbar^2}{\mu} f_c(\theta). \quad (3)$$

We can express θ the scattering angle in terms of momentum transfer, q as q=$2k\sin\theta/2$ where, k=$\sqrt{2\mu E/\hbar^2}$.

So that as θgoes from 0 to pi, q goes from 0 to $2k$.

Therefore,

$T_{fi}(\theta)$=$T_c(q)$=$\frac{-4\pi\eta\hbar^2 k}{q^2}\exp^{[-\imath\eta\ln(q^2/4k^2)+2\imath\sigma_0]}$

This is the t-matrix effective interaction in momentum space. It can be very nicely evaluated because it behaves smoothly from $q = 0$ to ∞

For the exchange terms however the $\theta \to \pi$-θ and the corresponding $\sin(\theta/2)$ changes to $\cos(\theta/2)$ leading to $q^2 \to (k^2 - q^2/4.)$. The

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corresponding expression for the symmetrizing term, $T_C^S(q)$ is,

$$T_C^S(q) \;=\; -\frac{4\pi\eta\hbar^2 k}{(k^2-q^2)/4} \exp[-\imath\eta(1-q^2/4k^2)+2\imath\sigma_0] \quad (4)$$

This seen to be blowing up when one goes from q=0 to ∞ for taking its Fourier transform. If one remembers the fact that we are interested for θ=0 to $\pi/2$ *i.e* q from o to $\sqrt{2}k$, in which region it behaves nicely. Therefore we fitted $T_C^S(q)$ by a function which nicely reproduces its behaviour between 0 and $\sqrt{2}k$ and outside $\sqrt{2}k$ it goes down to zero smoothly. We found that it can be nicely fitted by Saxon-Woods and its derivative form as follows.

$$T_C(q) = t_R(q) + \imath t_I(q) \qquad (5)$$

where,

$$t_R(q) = t_{SW}\{1+\exp[(q-q_R)/a_R]\}^{-1} + 4t_{SWD}\exp\{(q-q_D/a_D)\}\{1+\exp[(q-q_D)/a_D\}^{-2}$$

and

$$t_I(q) = t_{ISWD}\exp\{(q-q_{ID})/a_{ID}\} \{1+\exp[q-q_{ID})/a_{ID}]\}^{-2}$$

The various parameters have been worked out for Coulomb scattering at the C-C vertex for $^{16}O(C,2C)^4He$ reaction at various energies. Interesting trends are seen: the strengths decreasing with energy, the radii increasing with energy and the diffusivities increasing suddenly with energy but staying constant over a large stretch of energy. Fourier transform of these $T_C(q)$ leads to the t-matrix effective Coulomb interaction, $t_C(r)$ as a function of separation distance, r between the changed particles (see Fig.(3))

Symmetrized and antisymmetrized t-matrix effective Coulomb interactions can thus be worked out in simple forms.

Few of the numerous applications of the present formalism are in the X-ray production cross section from electron knockout, finding out the scattering wave functions at distances where nuclear interaction vanished, the Coulomb scattering pertaining to the lower energy astrophysics application etc.

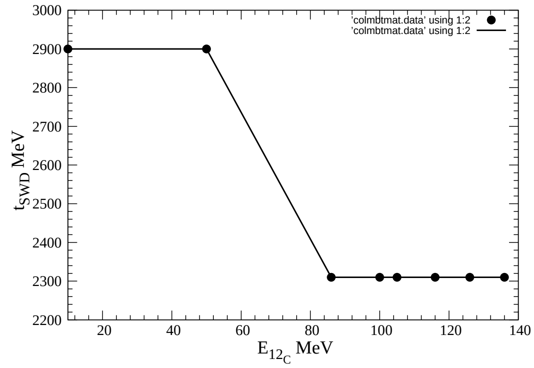

FIG. 2: Real depth of the derivative Saxon-Woods expansion of the exchange t-matrix $T_C^S(q)$ as a function of energy.

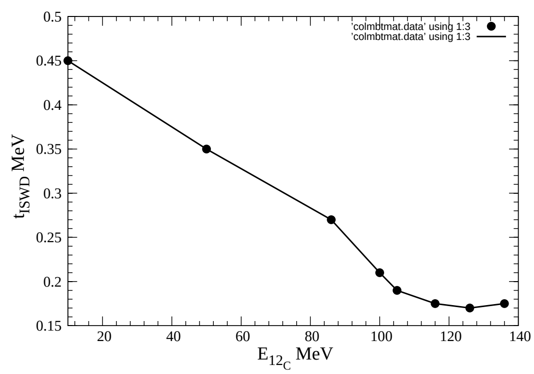

FIG. 3: Imaginary depth of the derivative Saxon-Woods expansion of the exchange t-matrix $T_C^S(q)$ as a function of energy.

References

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[2] M. R. C. McDowell and J. P. Coleman, Intr. Theo. Ion-Atom Coll. North Holland, 251(1970).