Application of the Lagrange-mesh technique to low energy direct breakup reactions

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Introduction

Study of Coulomb dissociation of weakly bound nuclei has emerged as a powerful tool in determining the cross-sections of astrophysiclly interesting radiative capture reactions and hence their rates. Therefore, the motivation arises in developing a fully quantum mechanical theory which can be applied to cases in which the valence nucleons are either neutron(s) or proton(s).

Formalism

We consider the reaction $a + t \rightarrow b + c + t$, where the projectile a breaks up into fragments b (charged) and c (charged/uncharged) in the Coulomb field of a target t. The triple differential cross section for the reaction is given by

$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_c} = \frac{2\pi}{\hbar v_a}\rho(E_b, \Omega_b, \Omega_c)\sum_{lm}|\beta_{lm}|^2 \quad (1)$$

Here v_a is the a-t relative velocity in the entrance channel and $\rho(E_b, \Omega_b, \Omega_c)$ the phase space factor appropriate to the three-body final state. β_{lm} is the reduced amplitude in post form of finite range distorted wave Born approximation, given by

$$\hat{l}\beta_{lm}(\mathbf{q}_b, \mathbf{q}_c; \mathbf{q}_a) = \iint d\mathbf{r}_1 d\mathbf{r}_i \chi_b^{(-)*}(\mathbf{q}_b, \mathbf{r})$$
$$\chi_c^{(-)*}(\mathbf{q}_c, \mathbf{r}_c) V_{bc}(\mathbf{r}_1)\phi_a^{lm}(\mathbf{r}_1)\chi_a^{(+)*}(\mathbf{q}_a, \mathbf{r}_i) \quad (2)$$

where, $\mathbf{q}_b$, $\mathbf{q}_c$ and $\mathbf{q}_a$ are the wave vectors of b, c, and a corresponding to Jacobi vectors $\mathbf{r}$, $\mathbf{r}_c$ and $\mathbf{r}_1$, respectively. V_{bc} is the interaction between b and c. $\phi_a^{lm}(\mathbf{r}_1)$ is the ground state wave function of the projectile with relative orbital angular momentum state l and projection m. $\chi^{(-)}$'s are the distorted waves for relative motions of b and c with respect to t and the center of mass(c.m.) of the b-t system, respectively, with ingoing wave boundary condition. $\chi^{(+)}(q_a, r_i)$ is the distorted wave for the scattering of the c.m. of projectile a with respect to the target with outgoing wave boundary condition.

The evaluation of β_{lm} is quite complicated as it involves a six-dimensional integral. Earlier attempts were by using zero range approximation(ZRA) [1]. This approximation reduces the six-dimensional integral to three dimensional one, but this restricts the relative motion between b and c in the projecile a to s - state only. Another effective approximation was the Local Momentum approximation [2], which reduces the six-dimensional β_{lm} into two three-dimensional integrals, one giving the structural inforamtion and the other involving the dynamics of the reaction which in turn can be analytically solved in terms of the Bremsstrahlung integral.

However in all these previous attempts the fragment 'c' was uncharged and that was crucial in our analytical evaluation. In the next section, we shall introduce the Lagrange-mesh technique which can not only arrive at analytical results, but also at a faster pace.

Lagrange-mesh technique

This is an approximate variational calculation method, with the simplicity of a mesh calculation [3] because of the use of a consistent Gauss quadrature. Use of the Lagrange orthonormal functions makes it very effective because no analytical evaluation of matrix elements is needed. One only need to evaluate the potential at mesh points and this leads to faster numerical convergence. The accuracy is

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exponential in the number of mesh points.

In this method a variational approximation of the wave function is given by an expansion in the Lagrange basis,

$$\psi(x) = \sum_{i=1}^{N} c_i f_i(x) \tag{3}$$

In this basis, the coefficients c_i have a simple physical interpretation,

$$c_i = \lambda_i^{1/2} \psi(x_i), \tag{4}$$

where the mesh points x_i and weights λ_i define an approximate quadrature rule of the Gauss type

$$\int_a^b g(x)dx \approx \sum_{i=1}^{N} \lambda_i g(x_i). \tag{5}$$

Lagrange functions are infinitely differentiable functions which satisfy the Lagrange conditions,

$$f_i(x_j) = \lambda_i^{1/2} \delta_{ij}, \tag{6}$$

for which the Gauss quadrature is exact for products $f_i(x)f_j(x)$. As a corollary of the exactness of the Gauss quadrature, Lagrange functions are orthonormal, since

$$\int_a^b f_i(x) f_j(x) dx = \sum_{k=1}^{N} \lambda_k f_i(x_k) f_j(x_k) = \delta_{ij}. \tag{7}$$

Also one can regularize [4, 5] the Lagrange basis, i.e. can multiply this by some convenient factor, without losing their high accuracy. This feature is helpful when one faces a singularity in potential, which is possible in number of potentials. The singularities destroy the accuracy of the Gauss quadrature and hence the Lagrange-mesh method. Another important property of the Lagrange bases is "scaling"and "mapping". Scaled functions are defined as

$$\hat{f}_j(x) = h^{-1/2} f_j(x/h). \tag{8}$$

They still verify the Lagrange condition (6). The interval of definition becomes (ha, hb).The scaling can be used to fit the mesh-point distribution to the physical problem. The scaling parameter h can be used as a variational parameter.

Mapping is a more general transformation,

$$\hat{f}_j(x) = [t'(x)]^{1/2} f_j[t(x/h)]. \tag{9}$$

It leads to a change of interval but also to a change of mesh.

Our plan is to expand the distorted waves in Eq.(2) in the Lagrange basis and present the first results in this context. We will try to show, how by using the Lagrange-mesh technique one can actually arrive at analytical simplifications of the problem and also as an upshot have much faster numerical convergences than previous attempts.

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