Parametric relationship between Properties of static limit and rotating equilibrium sequences of compact stars

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Compact stars originate from the supernovae, just two years after the discovery of neutron by Chadwick (1932), Baade and Zwick (1934) first proposed the existence of neutron stars. The spinning neutron stars are called pulsers. Until now, no compact star has ever been found to spin faster than PSR J1748-2446 spinning at 716Hz [1] and PSR B1937+214 rotating at the frequency 641 Hz [2]. However the recent discovery of XTE J1739-285 [3] suggests that it contains a CS spinning at 1122 rotation per second - more than 1.5 times faster than any other CS, but it is important to reconfirm this signal. Such frequencies are too low to affect significantly the structure of CSs with $M > 1M_{\odot}$, since the dense matter CSs have the keplerian (mass-shedding) frequencies larger than 1KHz. The main objective of this work is to investigate the suitable parametric relationship between combination of observable properties of static limit sequences with the observable properties of uniformally rotating equilibrium configurations. The relationship between keplerian angular frequency (upper limit of rotation of compact star) Ω_K, gravitational mass M, and radius R of a compact star; $\Omega_K = \Omega_K(M, R)$ is based on the equations of state at supranuclear density. Therefore, it is of great interest to investigate the recently proposed empirical relationship as $\Omega_K = C(M, R)(M/M_{\odot})^{1/2}(10km/R)^{3/2}$ [4–6], here C(M,R) is the coefficient which depends upon mass and radius. We also proposed an empirical relationship of δM_B with suitable combination of Ω_K and $R_{1.4}$ (radius of star having mass $1.4M_{\odot}$) where $\delta M_B = M^{stat}_{B,max} - M_{B,min}$, here $M^{stat}_{B,max}$ is maximum baryonic mass of nonrotating CS and $M_{B,min}$ is the minimum mass for the CS rotating 1122Hz frequency or period of 0.891 ms by using a representative set of EOSs. We employ representative set of 21 EOSs with maximum gravitational mass $M^{\rm stat}_{\rm max}$ of nonrotating star varying from $1.63M_{\odot}$ - $2.50M_{\odot}$. These EOSs are constructed using various theoretical models of nuclear dense matter. The chemical composition of these EOSs are varying from the nucleons to hyperons, kaons, and quarks in β-equilibrium. It is found that the calculated values of M^{stat}_{max} and R_{max}(radius of star at maximum mass M^{stat}_{max}) of nonrotating CSs have shown empirical relationship with Ω_K, The approximate empirical relationship for rotating CS with keplerian angular frequency in terms of maximum gravitational mass and radius of the nonrotating star is given by,

$$\Omega_K(10^4 s^{-1}) = a_o + a_1 \left(\frac{M_{max}}{M_{\odot}}\right)^{1/2} \left(\frac{10km}{R_{max}}\right)^{3/2}, \quad (1)$$

The best fit values of the parameters appearing in Eq. (1) are calculated by using the results of CS mass and radius for each EOS of the representative set as presented in Fig.1. The circles are for the selected set of EOSs and solid line is calculated by emplying the approximate emperical relation Eq. (1) by using the values of best fitted parameters. The values of parameters are; $a_o = -1.0 \times 10^3\ s^{-1}$ and $a_1 = 8.6 \times 10^3 s^{-1}$. In Fig.2 we plot $\delta M_{\rm B}$ as a function of keplerian angular frequency Ω_K in the left panel A, and moment of inertia I_{1122} for compact star uniformly rotating with frequency 1122Hz in the right panel B. It is clear from the Fig.2 that for a CS to be a superamassive ($\delta M_B < 0$) at rotation frequency 1122 Hz, should have the value of CS radius $R_{1.4} \geq$ 12.50 km, moment of inertia $I_{1122} \geq 2.5 \times 10^{45}$ gcm^2 , and the values of keplerian frequency $\Omega_K \lesssim 10^4 s^{-1}$. In Fig.3 we parametrize the δM_B with the appropriate combination of keplerian angular frequency and $R_{1.4}$ by the fol-

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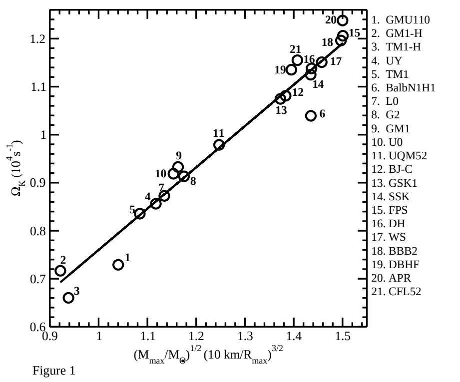

FIG. 1: Correlation between keplerian angular velocities calculated within general relativistic framework assuming uniform rotation with nonrotating mass-radius relationship $(M_{max}/M_{\odot})^{1/2}(10km/R_{max})^{3/2}$.

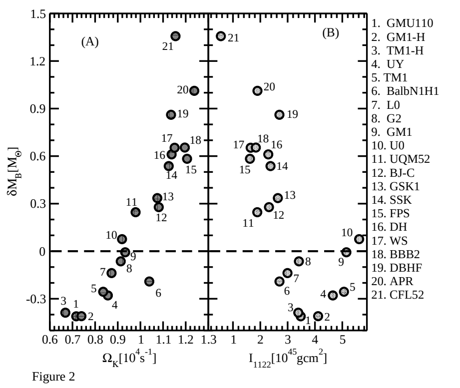

FIG. 2: Correlation of the $\delta M_{\rm B}$ with Ω_K in the left panel A, and with values of moment of inertia I_{1122} for compact star uniformly rotating with frequency 1122Hz in the right panel B.

lowing relation;

$$\frac{\delta M_B}{M_{\odot}} = b_o + b_1 \left(\frac{\Omega_K}{10^4 s^{-1}}\right)^{\alpha} \left(\frac{10km}{R_{1.4}}\right)^{\beta}. \quad (2)$$

Here $\alpha = 2.75$ and $\beta = 0.30$, are obtained by using the best fitting procedure. The values of parameters are $b_0 = -0.8$ and $b_1 = 0.9$. The combination of Ω_K and $R_{1.4}$ shows the reasonable good correlation with δM_B, whereas any

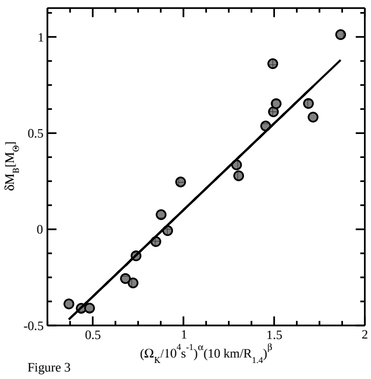

FIG. 3: Correlation between $\delta M_{\rm B}$ with $(\Omega_K/10^4 s^{-1})^{\alpha}(10km/R_{1.4})^{\beta}$. The circles are for the selected set of EOSs and solid line is calculated by employing the approximate empirical relation Eq. (2.)

suitable combination for properties of rotating star such as Ω_K with J_{1122} or I_{1122} are weakly correlated with δM_B.

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