

## Binding of Lambda hypernuclei within a relativistic mean field approach

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### Introduction

Current nuclear physics is focussed on exploring fundamental interaction between nucleon (N) and hyperon (Y) and basic properties of the hadronic system. The worldwide activity in doing many sophisticated experiments [1, 2] in these interdisciplinary fields shows the strong scientific interest in exploring the hypernuclear properties. Hypernuclear physics is of great importance in many branches of physics. Of particular interest is the understanding of strange [3] particles in baryonic matter, since many questions in heavy-ion physics, particle physics and astrophysics are related to the effect of strangeness (S) in nuclear matter. Moreover, the contribution of hyperons strongly influences the masses of neutron stars as well. In the past decade a considerable amount of spectroscopic information has been accumulated experimentally on the  $\Lambda^0$  ( $S = -1$ ) hypernuclei. The  $\Lambda^0$  separation energies ( $S_Y$ ) have been determined for the ground states of about 40  $\Lambda^0$  hypernuclei including several double- $\Lambda^0$  hypernuclei [4]. The strong  $YN$  and  $YY$  interaction are useful to explain the phase transition in the interior of the compact stars like strange quark stars, hybrid stars, hypercompact hyperon stars etc. composed of the strongly interacting densest matter in the universe undergoing the deconfinement phase transition. Therefore it is interesting to study effect of the addition of  $\Lambda$  hyperon(s) inside the core of normal nuclei which sheds some light on the interactions among hadrons. In this work, we es-

timate the binding of a  $\Lambda$ -hyperon to the normal nuclear core for several nuclei (e.g.  ${}_{\Lambda}^{16,17}O$ ,  ${}_{\Lambda}^{40,41}Ca$ ,  ${}_{\Lambda}^{56}Fe$ ,  ${}_{\Lambda}^{89}Y$ ,  ${}_{\Lambda}^{139}La$ ,  ${}_{\Lambda}^{208}Pb$  etc.) within a relativistic mean field (RMF) approach.

### RMF Approach

The RMF theory has proved to be successful as a framework for description of various facets of nuclear properties. The Dirac-Hartree equations for a finite spherical nucleus can be derived from an interacting RMF theory in which the nuclear force is produced by a virtual exchange of various mesons. The starting point of the RMF theory is the model due to Walecka [5] which describes the nucleons as Dirac spinors interacting by the exchange of several mesons: scalar mesons ( $\sigma$ ) couple to the nucleons ( $\psi$ ) through a Yukawa term  $\bar{\psi}\sigma\psi$  and produce strong interaction, isoscalar vector meson ( $\omega$ ) couple to conserved nucleon current  $\bar{\psi}\gamma_{\mu}\psi$  and cause a repulsion. In addition, there are isovector  $\rho$  mesons which couple to the isovector current and photons to produce the well-known electromagnetic interaction. To study the  $\Lambda$  hyperonic effects on finite nuclei we use the following Lagrangian density:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - M_N)\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma - m_{\sigma}^2\sigma^2) \\ & - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu} - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} \\ & + \frac{1}{2}m_{\rho}^2\vec{\rho}_{\mu}\cdot\vec{\rho}^{\mu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} + g_{\sigma}\bar{\psi}\psi\sigma \\ & - \bar{\psi}\gamma_{\mu}(g_{\omega}\omega^{\mu} + \frac{g_{\rho}}{2}\vec{\tau}\cdot\vec{\rho}^{\mu} + \frac{e}{2}A^{\mu}(1 + \tau_3))\psi \\ & - \frac{\kappa}{3!}(g_{\sigma}\sigma)^3 - \frac{\lambda}{4!}(g_{\sigma}\sigma)^4 + \frac{\zeta}{4!}(g_{\omega}^2\omega_{\mu}\omega^{\mu})^2 \\ & + \Lambda_v(g_{\rho}^2\vec{\rho}_{\mu}\cdot\vec{\rho}^{\mu})(g_{\omega}^2\omega_{\mu}\omega^{\mu}) + \mathcal{L}_h \quad (1) \end{aligned}$$

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where,

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (2)$$

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \quad (3)$$

$$\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu - g_\rho (\vec{\rho}_\mu X \vec{\rho}_\nu) \quad (4)$$

$$\mathcal{L}_h = \bar{\psi}_h [i\gamma_\mu \partial^\mu - M_h - g_\sigma^h \sigma - g_{\sigma^*}^h \sigma^* - g_\omega^h \gamma^0 \omega - g_\phi^h \gamma^0 \phi + \frac{f_\omega^h}{2M_h} \sigma^{0i} \partial_i \omega] \psi_h \quad (5)$$

We introduce a  $\Lambda$  hyperon in the system through the Lagrangian density given by Eq.(5) in which we treat  $\Lambda$  as a charge neutral isoscalar without any coupling to  $\rho$  and photon. The effects of  $\Lambda$  hyperon appears through the coupling with  $\sigma$  ( $g_{\sigma\Lambda}$ ) and  $\omega$  ( $g_{\omega\Lambda}$ ) mesons. We do not incorporate the contributions from  $\sigma^*$  and  $\phi$  for single  $\Lambda$ . The centre of mass energy correction is taken as  $E_{cm} = \frac{\langle F | \hat{P}_{total}^2 | F \rangle}{M_{total}} \approx \frac{0.75 \times 41}{A^{1/3}}$ . We solve the mean field equations by iterative method starting from the initial guess for the fields and then solve the Dirac equations using the derived meson fields. The pairing interaction for the nucleons are also considered here.

## Results and discussion

We consider  $\Lambda$  hyperon only for the calculation of hyperon binding energy inside a hypernucleus. The coupling for normal nucleons and meson mass are taken from FSUGold force. Recently, P. Roy Chowdhury *et al.* [4] have calculated the hyperon ( $\Lambda$ ,  $\Lambda\Lambda$ ,  $\Xi^{-,0}$  etc.) separation energy for some hypernuclei and thoroughly studied the effects of hyperons on the nuclei near drip line on the basis of phenomenological BWMH mass estimate. In comparison to the BWMH estimates ( $S_Y^{BWMH}$ ) [4], the calculated values in this work ( $S_Y^{RMF}$ ) are in better agreement with the measured values ( $S_Y^{EX}$ ) [1, 2, 4] for most of the hypernuclei given in Table-1.

## Summary

We calculate the binding energy of  $\Lambda$  hyperon inside a hypernucleus within a RMF approach using FSUGold parameter set. The numerical code has been developed for the single strange  $\Lambda$  hyperon. In this short report,

TABLE I: The calculated ( $S_Y^{RMF}$ ,  $S_Y^{BWMH}$ ) and measured ( $S_Y^{EX}$ ) values of  $\Lambda$  separation energy ( $S_Y$  in MeV) for some hypernuclei are given in the table. The binding energy per nucleon ( $-B/A$  in MeV), radius (fm) of neutron ( $r_n$ ) and proton ( $r_p$ ) distributions of the hypernuclei are given from RMF calculation.

${}^A_\Lambda Z$	-B/A	$S_Y^{RMF}$	$S_Y^{EX}$	$S_Y^{BWMH}$	$r_n$	$r_p$
${}^{16}_\Lambda O$	7.8	12.3	12.5	14.3	2.32	2.42
${}^{17}_\Lambda O$	8.2	12.5	13.6	14.6	2.40	2.42
${}^{28}_\Lambda Si$	8.2	17.0	16.0	17.7	2.75	2.83
${}^{32}_\Lambda S$	8.3	18.0	17.5	18.3	2.91	3.02
${}^{33}_\Lambda S$	8.4	18.4	18.0	18.4	2.98	3.02
${}^{40}_\Lambda Ca$	8.6	18.2	18.7	19.4	3.16	3.24
${}^{41}_\Lambda Ca$	8.8	18.3	19.2	19.4	3.20	3.25
${}^{51}_\Lambda V$	8.8	20.2	19.9	20.4	3.45	3.39
${}^{56}_\Lambda Fe$	8.8	20.7	21.0	20.7	3.54	3.51
${}^{89}_\Lambda Y$	8.8	22.4	22.1	22.4	4.42	4.32
${}^{139}_\Lambda La$	8.5	23.6	23.8	24.0	4.90	4.73
${}^{208}_\Lambda Pb$	8.0	25.5	26.5	25.3	5.62	5.42

we have shown our results only for twelve hypernuclei which are in better agreement with measured values than that using phenomenological BWMH formula. In future, a project on multi-strange hyperons will be taken to explore the influence of strangeness in the nuclear medium.

## Acknowledgments

The research work of P. Roy Chowdhury is sponsored by the UGC Grant (No.F.4-2/2006(BSR)/13-224/2008(BSR)) under Dr. D.S. Kothari Postdoctoral Fellowship Scheme.

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