The decay widths and the spectrum of charmonium

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Introduction

We have calculated the S wave charmonium spectrum and decays using a relativistic quark model. In our present work, for the confinement of the quarks we have made use of the relativistic harmonic model (RHM) [1]. For the confinement of gluons we have made use of the current confinement model (CCM) which was developed in the spirit of the RHM [2–4]. The confined gluon propagators (CGP) were derived in the CCM, which were then used to derive the confined one gluon exchange potential (COGEP) [5]. We have also calculated the meson decay constants, the leptonic decay width, three photon decay width and hadronic decay widths for the ground state charmonium.

The Model

In the RHM [1], quarks in a hadron are confined through the action of Lorentz scalar plus a vector harmonic oscillator potential,

$$V_c = \frac{1}{2}(1+\gamma_0)A^2r^2 + M \tag{1}$$

where γ_0 is the Dirac matrix, M is the quark mass and A^2 the confinement strength.

The quark-antiquark interaction potential is given by the COGEP. The central part of COGEP is [5]:

$$V_{COGEP}^{cent} = \frac{\alpha_s}{4}N^4\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j[D_0(r) + \frac{1}{(E+M)^2} [4\pi\delta^3(r) - c^4r^2D_1(r)][1-\frac{2}{3}\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j]], \tag{2}$$

where the first term is the residual Coulombic interaction and the second and the third are the chromomagnetic interaction leading to the hyperfine splittings.

The $q\bar{q}$ wave function for each meson state is expressed in terms of oscillator wave functions corresponding to the center of mass (CM) and relative coordinates. The oscillator quantum number for the CM wavefunctions are restricted to $N_{CM} = 0$. The Hilbert space of relative wavefunctions is truncated at radial quantum number $n = 5$. The Hamiltonian matrix is constructed in the basis states of $|N_{CM} = 0, L_{CM} = 0; n\ ^{2S+1}L_J >$ and diagonalised. The diagonal values give the masses of the ground and radially excited states.

The parameters used in our model are charm quark mass $m_c = 1.48$ GeV, $\alpha_s = 0.2$ and the oscillator size parameter $b_n(= 1/\Omega_n)$, which is 0.304 fm for triplet mesons and 0.379 fm for singlet mesons.

The decay constants of the pseudoscalar and vector mesons are given by the Van Royen and Weisskopf formula [6, 7]:

$$f^2_{P/V} = 12\frac{|\psi_{P/V}(0)|^2}{m_{P/V}}$$

The leptonic decay width for a n^3S_1 state to decay to a lepton pair is given by [6–8]:

$$\Gamma_{l^+l^-} = 16\pi\alpha^2e_q^2\frac{|\psi(0)|^2}{m_V^2}\left(1-\frac{16\alpha_s}{3\pi}\right)$$

The three photon decay ($^3S_1 \to \gamma\gamma\gamma$) for J/ψ is given by [9]:

$$\Gamma_{\gamma\gamma\gamma} = \frac{16(\pi^2-9)\alpha^3e_q^6|\psi(0)|^2}{3m_q^2}\left(1-\frac{12.6\alpha_s}{\pi}\right)$$

The decay width for $J/\psi \to ggg$ and $J/\psi \to \gamma gg$ are given respectively by [9]:

$$\Gamma_{ggg} = \frac{40(\pi^2-9)\alpha_s^3|\psi(0)|^2}{81m_q^2}\left(1-\frac{3.7\alpha_s}{\pi}\right)$$

$$\Gamma_{\gamma gg} = \frac{32(\pi^2-9)\alpha\alpha_s^2e_q^2|\psi(0)|^2}{9m_q^2}\left(1-\frac{6.7\alpha_s}{\pi}\right)$$

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Results and discussions

The masses of the singlet and triplet S-wave charmonium are listed in Table I in comparison with experiment [11]. Table II gives the mesonic decay constant, leptonic decay width, three-photon decay width and the hadronic decay width for J/ψ in comparison with experiment and other models. The results obtained are in good agreement with the experimental values.

TABLE I: Mass Spectrum (in MeV)

Meson	**Present**	**PDG[11]**	**[12]**
J/ψ	3097	3097	3100
$\psi(2S)$	3646	3686	3730
$\psi(3S)$	4102		4180
$\psi(4S)$	4687		4560
$\psi(5S)$	4892		
$\eta_c(1S)$	2980	2980	3000
$\eta_c(2S)$	3391	3637	3670
$\eta_c(3S)$	3725		4130
$\eta_c(4S)$	4169		

TABLE II: Decay Properties

Observable	**Present**	**Exp.**	**Others**
$f_P(\eta_c(1s))$	319 MeV	335 MeV	424 Mev[6]
$f_V(J/\psi)$	437 MeV	411 MeV	423 Mev[6]
$\Gamma_{e^+e^-}$	4.04 keV	5.55 keV	5.41 keV[10]
$\Gamma_{\gamma\gamma\gamma}$	0.7 eV	$1.2 \times 10^{-5\,a}$	
Γ_{ggg}	77.2 keV	$64.1 \pm 1.0\%^a$	59.5 keV[10]
$\Gamma_{\gamma gg}$	5.2 keV	$8.8 \pm 0.5\%^a$	5.7 keV[10]

[a]Branching Fraction

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