Light meson radiative transitions in non-relativistic quark model

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Introduction

Mesons exhibit several different decay modes such as leptonic, semileptonic, hadronic, radiative, two photon decays and so on. Depending on the decay products and the force involved we classify the various modes. Goal of elementary particle theory is to calculate their lifetimes and branching ratios. Using phenomenological models, we have seen, one can obtain meson mass spectra. However we know that the description of the spectra is a necessary but not a sufficient condition for aiming at a good explanation of non perturbative QCD. Hence one needs other observables which rely essentially on the same dynamical operators like transitions between various states, in order to test the model under study. So, one of the tests for the success of any theoretical model for mesons is the correct prediction of their decay rates. For this, one can take different possible decays allowed by conservation laws and governed by one of the three fundamental forces: strong, electromagnetic and weak. Here we consider the E1 and M1 transition widths for light mesons. The electric dipole term is responsible for the transition between the S and P states with the same spin S of the quark pair, while the M1 term describes the transitions between S=1 and S=0 states with the same orbital momentum L.

The rate for transitions from a $^3S_1$ state to ^3P_J state [1] is given by,

$$\Gamma_{(^3S_1 \to \gamma ^3P_J)} = (2J+1)\frac{4}{27} e_q^2 \alpha k_0^3 |I_{PS}|^2 ,$$

where k_0 is the energy of the emitted photon, e_q is the charge of the quark, α is the fine structure constant and I_{PS} is the radial overlap integral which has the dimension of length.

$$I_{PS} = \langle P|r|S\rangle = \int_0^\infty r^3 R_P(r) R_S(r)dr$$

with $R_{S,P}(r)$ being the normalised radial wave functions for the corresponding states. The transition from ^3P_J levels to a $^3S_1$ level is described by the expression for the rate

$$\Gamma_{(^3P_J \to \gamma ^3S_1)} = \frac{4}{9} e_q^2 \alpha k_0^3 |I_{SP}|^2$$

For transitions $^1P_1$ --> $^1S_0$ the same above expression is used to calculate the rate.

The allowed M1 transitions are essentially $^3S_1$ --> $^1S_0$ and $^1S_0$ --> $^3S_1$. The rate for transitions from a $^3S_1$ state to $^1S_0$ state is given by

$$\Gamma_{(n^3S_1 \to \gamma m ^1S_0)} = \frac{4}{3m^2} e_q^2 \alpha k_0^3 |I_{mn}|^2 ,$$

where I_{mn} is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states and m is the mass of the quark.

$$I_{mn} = \int_0^\infty r^2 R_{nS}(r) R_{mS}(r)dr$$

For transitions from $^1S_0$ state to $^3S_1$ state the following expression for the rate is used

$$\Gamma_{(n^1S_0 \to \gamma m ^3S_1)} = \frac{4}{m^2} e_q^2 \alpha k_0^3 |I_{mn}|^2$$

The above radiative decay widths are calculated in the frame work of non relativistic quark model. The expressions are derived for the mesons having the same flavour wave functions ($n\bar{n}$). These expressions are derived from the general expressions given in [2]. For radiative decays involving mesons with mixed flavours such as charged decays of ρ

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($(u\bar{u} - d\bar{d})/\sqrt{2}$) into $\pi\gamma$ the charge contents are calculated as in references [2, 3].

Results and discussions

We have calculated the E1 and M1 transition widths for light meson states. These transitions are well-known and are reported in PDG [4]. We have studied the transitions allowed by long wavelength approximation. They are $^3P_J \to {}^3S_1$ (E1 transitions); $^3S_1 \to {}^1S_0$ and $^1S_0 \to {}^3S_1$ (M1 transitions). Also we have studied a particular E1 transition corresponding to the decay of $^1P_1 \to {}^1S_0$.

In non relativistic treatment energy of the photon (k_0) is equal to the energy difference between the resonances. And the term $E_b(k_0)/m_a$ (here $E_b(k_0)$ is the energy of final resonance at k_0 and m_a is the mass of initial resonance) in non relativistic phase space is equal to unity. In our calculations the experimental values of the meson masses have been used. Our results are shown in the table 1.

Conclusions

Radiative decay widths have been investigated in non relativistic quark model for light mesons. The decay widths of some of the transitions agree with experimental values. We find the rest far from good agreement with experimental values. Better results could be obtained if relativistic phase space is used.

Table 5. 23 Radiative decay widths of light mesons

Transition	Expl. value Γ(keV)	Calculated Γ (keV)
$^3S_1 \to {}^1S_0$		
$\rho^+ \to \pi^+\gamma$	67.82 ± 7.55	545.57
$\rho^0 \to \pi^0\gamma$	102.48 ± 25.69	556.03
$\rho^0 \to \eta\gamma$	36.18 ±13.57	71.62
$\omega \to \pi^0\gamma$	714.85 ± 42.74	5343.2
$\omega \to \eta\gamma$	5.47 ± 0.84	9.44
$\phi(1020) \to \eta\gamma$	55.82 ± 2.73	255.22
$\phi(1020) \to \eta'(958)\gamma$	0.53 ± 0.31	0.29
$K^*(892)^0 \to K^0\gamma$	116.15 ± 10.19	372.63
$K^*(892)^+ \to K^+\gamma$	50.29 ± 4.66	252.23
$^1S_0 \to {}^3S_1$		
$\eta'(958) \to \rho^0\gamma$	61.31 ± 5.51	260.96
$\eta'(958) \to \omega\gamma$	6.11 ± 0.78	23.38
$^3P_J \to {}^3S_1$		
$f_1(1285) \to \rho^0\gamma$	1296 ± 295.2	1138.71
$^1P_1 \to {}^1S_0$		
$b_1(1235) \to \pi^+\gamma$	227.2 ± 58.6	780.21

References

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