Bottomonium spectrum in a relativistic quark model

K. B. Vijaya Kumar,* Bhaghyesh, and Antony Prakash Monteiro
Department of Physics, Mangalore University, Mangalagangothri - 574199, INDIA

Introduction

In this work, we have calculated the S wave bottomonium spectrum and decays using a relativistic quark model. In our present work, for the confinement of the quarks we make use of the relativistic harmonic model (RHM), which has been successful in explaining the properties of light hadrons [1]. For the confinement of gluons we have made use of the current confinement model (CCM) which was developed in the spirit of the RHM [2, 3, 5]. The confined gluon propagators (CGP) were derived in the CCM, which were then used to derive the confined one gluon exchange potential (COGEP) [6]. We have also calculated the meson decay constants, the leptonic decay width, and the hadronic decay widths for the ground state bottomonium.

The Model

In the RHM [4], quarks in a hadron are confined through the action of Lorentz scalar plus a vector harmonic oscillator potential,

$$V_c = \frac{1}{2}(1+\gamma_0)A^2r^2 + M \qquad (1)$$

where γ_0 is the Dirac matrix, M is the quark mass and A^2 the confinement strength.

The quark-antiquark interaction potential is given by the confined one-gluon exchange potential (COGEP). The central part of COGEP is given by [6]:

$$V^{cent}_{COGEP} = \frac{\alpha_s}{4}N^4\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j[D_0(r) + \frac{1}{(E+M)^2} [4\pi\delta^3(r) - c^4r^2D_1(r)][1-\frac{2}{3}\boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j]], \qquad (2)$$

where the first term is the residual Coulombic interaction and the second and the third are the chromomagnetic interaction leading to the hyperfine splittings.

The $q\bar{q}$ wave function for each meson state is expressed in terms of oscillator wave functions corresponding to the center of mass (CM) and relative coordinates. The oscillator quantum number for the CM wavefunctions are restricted to $N_{CM} = 0$. The Hilbert space of relative wavefunctions is truncated at radial quantum number $n = 5$. The Hamiltonian matrix is constructed in the basis states of $|N_{CM} = 0, L_{CM} = 0; n\ {}^{2S+1}L_J>$ and diagonalised. The diagonal values give the masses of the ground and radially excited states.

The parameters of the RHM are the mass of bottom quark (m_b) and the oscillator size parameter $b_n(= 1/\Omega_n)$. In our model, m_b = 4.82 GeV, $\alpha_s = 0.2$. The value of b is found to be 0.24 fm.

The charge radii is calculated using [7]:

$$< r^2 >^{1/2} = \left[\int_0^\infty |R_{n0}(r)|^2 r^2 r^2\, dr\right]^{1/2}$$

The decay constants of the pseudoscalar and vector mesons are given by the Van Royen and Weisskopf formula [8, 9]:

$$f^2_{P/V} = 12\frac{|\psi_{P/V}(0)|^2}{m_{P/V}}$$

The leptonic decay width is given by [8–10]:

$$\Gamma_{l^+l^-} = 16\pi\alpha^2e_q^2\frac{|\psi(0)|^2}{m_V^2}\left(1-\frac{16\alpha_s}{3\pi}\right)$$

The decay width for $\Upsilon \to ggg$ and $\Upsilon \to \gamma gg$ are given respectively by [11]:

$$\Gamma_{ggg} = \frac{40(\pi^2-9)\alpha_s^3|\psi(0)|^2}{81m_q^2}\left(1-\frac{4.9\alpha_s}{\pi}\right)$$

$$\Gamma_{\gamma gg} = \frac{32(\pi^2-9)\alpha\alpha_s^2e_q^2|\psi(0)|^2}{9m_q^2}\left(1-\frac{7.4\alpha_s}{\pi}\right)$$

*Electronic address: kbvijayakumar@yahoo.com

Results and discussions

The obtained masses for the singlet and triplet S-wave bottomonium are listed in Table I in comparison with experiment [12]. The obtained values are in good agreement with the experimental results. The calculated charge radii is also given in Table I. The meson decay constants for the ground state are calculated for vector and pseudoscalar mesons. We have also caculated the leptonic decay width and the hadronic decay widths of $\Upsilon(1S)$. The results are given in Table II in comparison with experimental and other theoretical values. The values obtained are in good agreement with the experimental results.

TABLE I: Mass(in MeV) and Charge radii(in fm)

Meson	**Mass**	**PDG[12]**	**[13]**	**C Radii**	**[7]**
$\Upsilon(1S)$	9460	9460	9460	0.294	0.186
$\Upsilon(2S)$	9862	10023	10020	0.449	0.400
$\Upsilon(3S)$	10193	10355	10390	0.563	0.707
$\Upsilon(4S)$	10699	10579	10680	0.657	1.165
$\Upsilon(5S)$	10978		10930	0.740	1.850
$\eta_b(1S)$	9455	9391	9410		
$\eta_b(2S)$	9858		10000		
$\eta_b(3S)$	10190		10370		
$\eta_b(4S)$	10696		10660		

TABLE II: Decay Properties

Property	**Present**	**Exp.**	**Others**
$f_P(\eta_c(1s))$	356 MeV		599 Mev[8]
$f_V(J/\psi)$	356 MeV	708 MeV	665 Mev[8]
$\Gamma_{e^+e^-}$	0.324 keV	1.340 keV	1.35 keV[10]
Γ_{ggg}	10.2 keV	$81.7 \pm 0.7\%^a$	
$\Gamma_{\gamma gg}$	0.23 keV	$2.21 \pm 0.22\%^a$	

[a]Branching Fraction

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