Charmonium and bottomonium radiative transitions in non-relativistic quark model

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Introduction

We consider in this paper the E1 and M1 transition widths for heavy quarkonium namely charmonium and bottomonium states. The electric dipole term is responsible for the transition between the S and P states with the same spin S of the quark pair, while the M1 term describes the transitions between S=1 and S=0 states with the same orbital momentum L.

The rate for transitions from a $^3S_1$ state to ^3P_J state [1] is given by,

$$\Gamma_{(^3S_1 \to \gamma^3 P_J)} = (2J+1)\frac{4}{27} e_q^2 \alpha k_0^3 |I_{PS}|^2 ,$$

where k_0 is the energy of the emitted photon, e_q is the charge of the quark, α is the fine structure constant and I_{PS} is the radial overlap integral which has the dimension of length.

$$I_{PS} = \langle P|r|S\rangle = \int_0^\infty r^3 R_P(r) R_S(r) dr$$

with $R_{S,P}(r)$ being the normalised radial wave functions for the corresponding states. The transition from ^3P_J levels to a $^3S_1$ level is described by the expression for the rate

$$\Gamma_{(^3P_J \to \gamma^3 S_1)} = \frac{4}{9} e_q^2 \alpha k_0^3 |I_{SP}|^2$$

For transitions $^1P_1$ --> $^1S_0$ the same above expression is used to calculate the rate.

The allowed M1 transitions are essentially $^3S_1$ --> $^1S_0$ and $^1S_0$ --> $^3S_1$. The rate for transitions from a $^3S_1$ state to $^1S_0$ state is given by

$$\Gamma_{(n^3S_1 \to \gamma m^1 S_0)} = \frac{4}{3m^2} e_q^2 \alpha k_0^3 |I_{mn}|^2 ,$$

where I_{mn} is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states and m is the mass of the quark.

$$I_{mn} = \int_0^\infty r^2 R_{nS}(r) R_{mS}(r) dr$$

For transitions from $^1S_0$ state to $^3S_1$ state the following expression for the rate is used

$$\Gamma_{(n^1S_0 \to \gamma m^3 S_1)} = \frac{4}{m^2} e_q^2 \alpha k_0^3 |I_{mn}|^2$$

The above radiative decay widths are calculated in the frame work of non relativistic quark model. The expressions are derived for the mesons having the same flavour wave functions ($c\bar{c}$ and $b\bar{b}$).

Results and discussions

We have calculated the E1 and M1 transition widths for charmonium and bottomonium states. These transitions are well-known and are reported in PDG [2]. We have studied the transitions allowed by long wavelength approximation. They are ^3P_J --> $^3S_1$ and $^3S_1$ --> ^3P_J (E1 transitions); $^3S_1$ --> $^1S_0$ and $^1S_0$ --> $^3S_1$ (M1 transitions). Also we have studied a particular E1 transition corresponding to the decay of $^1P_1$ --> $^1S_0$.

In non relativistic treatment energy of the photon (k_0) is equal to the energy difference between the resonances. And the term $E_b(k_0)/m_a$ (here $E_b(k_0)$ is the energy of final resonance at k_0 and m_a is the mass of initial resonance) in non relativistic phase space is equal to unity. In our calculations the experimental values of the meson masses have been used. Our results are shown in the tables 1-2. In the table 2, we have not included the decay $\Psi(2S)$ --> $\eta_c(1S)\ \gamma$ since the overlap integral vanishes for the

2S --> 1S transition. This decay is possible due to the relativistic effects which have not been considered in the present work.

Conclusions

Radiative decay widths have been investigated in non relativistic quark model for heavy mesons. The decay widths of some of the transitions agree with experimental values. The results obtained indicate that better results could be obtained if relativistic phase space is used.

Table1 Radiative decay widths of bottomonium states

Transition	Expl. value Γ(keV)	Calculated Γ (keV)
$^3S_1$ -->^3P_J		
$\Upsilon(2S) \to \chi_{b0}(1P)\gamma$	1.22 ± 0.16	1.20
$\Upsilon(2S) \to \chi_{b1}(1P)\gamma$	2.21 ± 0.22	1.87
$\Upsilon(2S) \to \chi_{b2}(1P)\gamma$	2.29 ± 0.23	1.90
$\Upsilon(3S) \to \chi_{b0}(2P)\gamma$	1.2 ± 0.16	1.03
$\Upsilon(3S) \to \chi_{b1}(2P)\gamma$	2.56 ± 0.34	1.67
$\Upsilon(3S) \to \chi_{b2}(2P)\gamma$	2.66 ± 0.41	1.83
^3P_J --> $^3S_1$		
$\chi_{b0}(1P) \to \Upsilon(1S)\gamma$	seen	79.97
$\chi_{b1}(1P) \to \Upsilon(1S)\gamma$	seen	100.74
$\chi_{b2}(1P) \to \Upsilon(1S)\gamma$	seen	115.39
$\chi_{b0}(2P) \to \Upsilon(1S)\gamma$	seen	40.16
$\chi_{b0}(2P) \to \Upsilon(2S)\gamma$	seen	19.01
$\chi_{b1}(2P) \to \Upsilon(1S)\gamma$	seen	43.86
$\chi_{b1}(2P) \to \Upsilon(2S)\gamma$	seen	26.01
$\chi_{b2}(2P) \to \Upsilon(1S)\gamma$	seen	46.05
$\chi_{b2}(2P) \to \Upsilon(2S)\gamma$	seen	30.63

Table 2 Radiative decay widths of charmonium states

Transition	Expl. value Γ(keV)	calculated Γ (keV)
	$^3S_1$ --> ^3P_J	
Ψ(2S) --> χ_{c0}(1P)γ	25.76 ± 3.81	25.94
Ψ(2S) --> χ_{c1}(1P)γ	24.10 ± 3.49	20.96
Ψ(2S) --> χ_{c2}(1P)γ	21.61 ± 3.28	14.32
	^3P_J --> $^3S_1$	
χ_{c0}(1P) --> J/ Ψ(1S) γ	92.40 ± 41.52	188.63
χ_{c1}(1P) --> J/ Ψ(1S) γ	240.24 ± 40.73	416.23
χ_{c2}(1P) --> J/ Ψ(1S) γ	270.00 ± 32.78	567.26
	$^1P_1$ --> $^1S_0$	
h_c--> η_c(1S) γ	seen	954.82
	$^3S_1$ --> $^1S_0$	
J/ Ψ(1S) --> η_c(1S) γ	1.13 ± 0.35	5.14

References

[1] Kwong W. and Rosner J. L. *Phys. Rev*. D **38,** 279 (1988).
[2] Amsler C et al. (Particle Data Group) *Phys. Lett*. B **667** 1 (2008) and its online update .