Charge, Flux Screening and Confinement in Dual QCD

H. Nandan[1],* N.M. Bezares-Roder[2], and H.C. Chandola[3]
[1] *Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi-110 025, INDIA*
[2] *Institut für Theoretische Physik, Universität Ulm, Ulm-890 69, GERMAY and*
[3] *Department of Physics, Kumaun University, Nainital-263 001, INDIA*

Introduction

The condensation of magnetically charged topological objects (viz. monopoles and dyons) in a way analogous to the Cooper pair condensation of electric charges in conventional superconductivity plays an important role in QCD to explain confinement and other closely related issues [1, 2]. In such dual superconductor model of QCD, the topological interaction is spanned over the different vacuum expectation values of complex scalar monopole/dyon field (ϕ) at different length scales in the spontaneously broken phase of symmetry [1]-[4]. A non-vanishing vacuum expectation value (VEV) of the scalar field in the ground state of such vacuum leads to the mass acquisition of the dual gauge field and thereby pushes the QCD vacuum in superconducting phase [3, 4].
In this paper, we study the screening (charge and flux) effects led by the magnetic condensation of the QCD vacuum along with their enlightening impacts on the quark confinement.

Lagrangian and Field Equations

We consider the following $U(1)$ Lagrangian with quarks [4, 5],

$$\pounds \;=-\,\frac{1}{4}\,\tilde{G}_{\mu\nu}\,\tilde{G}^{\mu\nu}+\Big|(\partial_\mu+i\,\tilde{g}\,\tilde{C}_\mu)\,\phi\;\Big|^2$$

$$+\bar{\psi}\,\gamma^\mu(i\,\partial_\mu+\tilde{g}\,\tilde{C}_\mu)\,\psi-m\,\psi\bar{\psi}\;-V(\phi^*\phi), \quad (1)$$

where $\tilde{G}_{\mu\nu}=\partial_\mu\tilde{C}_\nu-\partial_\nu\tilde{C}_\mu$ is the field strength corresponding to the dual gauge field $\tilde{C}_\mu$. Here ψ is the quark field with m as free-quark mass.

*Electronic address: hnandan.ctp@jmi.res.in

The effective potential in equation (1) has the following form,

$$V(\phi^*\phi)=\Omega\,(\,\phi^*\phi-\eta^2)^2, \qquad (2)$$

where $\eta^2\;=<\phi^*\phi>_0$ is the vacuum expectation value of ϕ. The potential (2) at $\phi=\eta$ gives the ground-state field configuration and captures the essential physics for the symmetry breakdown. The different field equations are then derived in the following form,

$$\partial^\nu\tilde{G}_{\mu\nu}+i\tilde{g}(\phi^*\overleftrightarrow{\partial_\mu}\phi)-\tilde{g}\bar{\psi}\gamma_\mu\psi-\tilde{m}^2\,\tilde{C}_\mu=0, \quad (3)$$

$$(\partial_\mu\;+\;i\,\tilde{g}\tilde{C}_\mu)^2\phi\;-2\Omega\,(\,|\phi|^2-\eta^2)\,\phi\;=\;0\,, \quad (4)$$

$$(\,i\gamma_\mu\,\partial^\mu+\tilde{g}\gamma^\mu\tilde{C}_\mu-m\,)\,\psi=0\,, \qquad (5)$$

where $\tilde{m}$ represents the mass of the dual gauge field which arises as an immediate consequence of the symmetry breakdown.

Screening Effects and Confinement Scenario

The total colour charge from equation (3) may be calculated as follows [5],

$$Q_c=\tilde{m}^2\int d^3x\,\tilde{C}_0+\tilde{g}\int d^3x\,\psi^\dagger\psi. \qquad (6)$$

Since all the observed hadrons are colour singlets and therefore the total colour electric charge of any quark system must vanish. The temporal gauge degrees of freedom for a single quark system leads to the following necessary condition for the total charge screening,

$$\tilde{m}^2\,\tilde{C}_0\;\gg\tilde{g}\,\psi^\dagger\psi. \qquad (7)$$

The equation (7) therefore puts a constraint on the quark system and $\tilde{C}_0$ plays a role of screening potential [5]. As such QCD vacuum having non-zero equilibrium values cannot be achieved by just using the notions of perturbative techniques where the equilibrium values of all the the fields is considered to be zero. It is therefore only possible to generate a vacuum screening current in the non-perturbative sector of QCD having the non-vanishing VEV of scalar field with the condition given by (7). These vacuum screening currents give mass to the dual gauge field and leads to a desired onset for the colour confinement. In order to find the axial symmetric confined solution of the field equations, we consider the following cylindrically symmetric ansatz,

$$\left.\begin{aligned} \tilde{\mathbf{C}} = -\, e_\theta\, \tilde{C}(\rho),\;\; \tilde{C}_0 = C_0(\rho) \\ \phi(\rho) = \zeta(\rho)\, \exp\left(i\, n\theta\right) \end{aligned}\right\}, \quad (8)$$

where $n = \pm 1, \pm 2, \ldots$, which measures the magnitude of the colour electric flux. In asymptotic regime for large ρ, the scalar field $|\phi| = \eta$ with $\zeta \to \eta$ plays the role of an order parameter like the Cooper wave function does in the Ginzburg-Landau theory of conventional superconductivity. The field equations corresponding to the colour electric and magnetic fields are then written as follows,

$$\frac{d\tilde{E}}{d\rho} - \frac{2n\tilde{g}}{\rho}\,\zeta^2 + \tilde{m}^2\,\tilde{C} = 0\,, \quad (9)$$

$$\frac{d\tilde{H}}{d\rho} - \frac{1}{\rho}\,\frac{d\tilde{C}_0}{d\rho} - \tilde{m}^2\,\tilde{C}_0 = 0\,, \quad (10)$$

with (7) and the following condition,

$$\tilde{m}^2\,\rho\,\tilde{C} - 2n\,\tilde{g}\,\zeta^2 \gg \rho\,(\psi^\dagger\,\boldsymbol{\gamma}\,\psi). \quad (11)$$

The electric and magnetic fields in equations (9) and (10) are defined respectively as,

$$\tilde{E} = -\frac{1}{\rho}\,\frac{d}{d\rho}\,(\,\rho\,\tilde{C})\,; \qquad \tilde{H} = \frac{d\tilde{C}_0}{d\rho}. \quad (12)$$

The solution of the equations (9) and (10) are exponentially approachable with some nonleading terms in the asymptotic limit [4]. For instance, the colour electric field at a particular coupling evolves generally as follows,

$$\tilde{E} \to C\,\sqrt{\frac{\eta}{\rho}}\;e^{-\,\tilde{m}\rho} + \cdots\cdots, \quad (13)$$

for $\rho \to \infty$ and here C is a constant. The equation (13) clearly indicates that the colour electric flux would screen out in the vacuum up to a finite depth $(1/\tilde{m})$ at large distances which determines the magnitude of the dual Meissner effect responsible for the colour confinement.

Conclusions

It is shown that the total charge and the colour flux screening in dual QCD vacuum in the background of the magnetic condensation are ultimately responsible for a dual Meissner effect and hence confinement. In fact, the temporal degrees for freedom of the dual gauge field (i.e. $\tilde{C}_0$) plays a role of screening potential in strong coupling regime which is responsible for the total charge screening. The equations (7) and (13) represent the conditions concerning the charge and flux screening respectively. These screening effects, in turn, guarantees the confined states of quarks (i.e. a flux tube like structure) and the formation of vortices in QCD vacuum. However, to understand the strength of confinement properly, it is necessary to calculate the exact energy contents associated to such bound states.

Acknowledgments

HN is thankful to the University Grants Commission, New Delhi, India for financial support under the UGC–Dr. D.S. Kothari post-doctoral fellowship programme.

References

[1] Y. Nambu, Phys. Rev. D **10**, 4282 (1974).
[2] S. Mandelstam, Phys. Rep. C **29**, 245 (1976).
[3] H. Nandan, H. C. Chandola and H. Dehenen, Int. J. Theor. Phys. **44**, 457 (2005).
[4] H. Nandan, T. Anna and H.C. Chandola, Euro Phys. Letts. **67**, 746 (2004).
[5] K. Shima, IL Nuovo Cimento **44**, 163 (1978).