N and Δ resonances in Hypercentral quark model

Kaushal Thakkar[1],* Ajay Majethiya[2], and P. C. Vinodkumar[1]
[1]*Department of Physics Sardar Patel University Vallabh Vidyanagar 388 120, Gujarat, INDIA and*
[2]*Kalol Institute of Technology and Research centre, Kalol,Gujarat, INDIA*

Introduction

The mass ordering of the excited nucleons has been a puzzle not only for traditional constituent quark models but also for other QCD based models. One of the famous problem is a failure of explaining a state of the first positive parity resonance of the nucleon $N^*(1440)$, so called "Ropper resonance." The chiral quark model suggests the importance of pseudoscalar meson (π, k, η) exchange potentials in the predictions of the excited baryons [1, 2]. In this paper, we make an attempt to study the N-Δ low lying states based on hyper central model with a three body potential of the form hyper Coulomb plus power potential($hCPP_\nu$) expressed in terms of the hyper radius (x) for the confinement part of the Hamiltonian [3]. We also incorporate the two body gluon and pseudoscalar meson exchange contributions separately.

Methodology

The Hamiltonian of baryonic systems in hypercentral model can be written as

$$H = \frac{P_\rho^2}{2\,m_\rho} + \frac{P_\lambda^2}{2\,m_\lambda} + V(\rho,\lambda) = \frac{P_x^2}{2\,m} + V(x) \quad (1)$$

For the present study, we consider the hyper central potential $V(x)$ as [3]

$$V(x) = V_{conf}\,(x) + V_{spin}\,(r_{ij}) + V_\gamma\,(r_{ij}) \quad (2)$$

Where, $V_{conf}\,(x) = -\frac{\tau}{x} + \beta x^\nu$ and τ of hyper Coulomb part is given by $\tau = \frac{2}{3}\alpha_s$, where α_s, the strong running coupling constant is considered here as 1.0 for the study of N-Δ system. We fix β, the strength of the confining term to reproduce the spin average masses of the 1S (1085 MeV), 2S (1520 MeV) and 3S (1815 MeV) positive parity states for each choices of potential index ν $(0.5 \le \nu \le 2.0)$. The spin dependent part of the three body interaction is taken as

$$V_{spin}(r_{ij}) = -\frac{1}{4}\alpha_s \sum_{i<j} \frac{\vec{\sigma_i}\cdot\vec{\sigma_j}}{6 m_i m_j} \vec{\lambda_i}\cdot\vec{\lambda_j}\ \delta^3(r_{ij}) \quad (3)$$

and the pseudoscalar interaction V_γ with $\gamma = \pi, \eta$ is given by [2]

$$V_\gamma(r_{ij}) = \frac{g_\gamma^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\gamma^2 \frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}} - \Lambda_\gamma^2 \frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}}\right\} \quad (4)$$

Following [2], we employ the same parameters of the meson exchange potentials and adopt the same linear scaling prescription for Λ_γ.

The energy eigen value corresponds to the Hamiltonian with the confining part of the potential are obtained by assuming a six dimensional trial radial wave function given by [3, 4]

TABLE I: Quark model parameter.

m_u,m_d MeV	μ_π MeV	μ_η MeV	Λ_0 MeV	κ	$\frac{g_8^2}{4\pi}$
338	139	547	566	0.81	0.67

$$\psi_{\omega\gamma} = \left[\frac{(\omega-\gamma)!(2g)^6}{(2\omega+5)(\omega+\gamma+4)!}\right]^{\frac{1}{2}} (2gx)^\gamma \times e^{-gx} L_{\omega-\gamma}^{2\gamma+4}(2gx) \quad (5)$$

The six dimensional wave function parameter g is determined by virial theorem. While for the two body interaction part of the Hamiltonian containing the one gluon exchange and the meson exchanges, we consider the three dimensional equivalent of Eqn. 5 with the wave function parameter $g_{3d} = \frac{g_{6d}}{n}$ MeV, where $n = 1, 2, 3, ...$ so as to match with the three

*Electronic address: kaushal21185@yahoo.co.in

TABLE II: First few low lying positive parity states of N and Δ in MeV .

Baryon	J^P	ν	V_{SS}	V_π	V_η	Mass	Expt. [5]	other
N(938)	$\frac{1}{2}^+$	0.50	-40.10	-19.54	0.60	1025.97	938-940	939 [6]
		1.00	-70.30	-31.00	1.03	984.74		936 [7]
		1.50	-95.39	-39.48	1.37	951.50		
		2.00	-115.95	-45.94	1.63	924.74		
Δ(1232)	$\frac{3}{2}^+$	0.50	40.10	19.54	-0.60	1144.03	1231-1233	1240 [6]
		1.00	70.30	31.00	-1.04	1185.26		1237 [7]
		1.50	95.39	39.48	-1.37	1218.50		
		2.00	115.95	45.94	-1.63	1245.26		
N(1440)	$\frac{1}{2}^+$	0.50	-29.02	-8.62	0.36	1482.72	1440±30	1459 [6]
		1.00	-61.62	-13.98	0.62	1445.02		1456 [7]
		1.50	-87.98	-17.37	0.79	1415.44		
		2.00	-109.75	-19.82	0.92	1391.35		
Δ(1600)	$\frac{3}{2}^+$	0.50	29.02	8.62	-0.36	1557.25	1625±75	1718 [6]
		1.00	61.62	13.98	-0.62	1594.97		1639 [7]
		1.50	87.98	17.37	-0.79	1624.37		
		2.00	109.75	19.82	-0.92	1648.65		
N(1710)	$\frac{1}{2}^+$	0.50	-14.81	-3.60	0.16	1796.74	1710±30	1776 [6]
		1.00	-30.68	-5.60	0.26	1778.98		
		1.50	-44.08	-6.92	0.32	1764.33		
		2.00	-55.22	-7.87	0.37	1752.29		
Δ(1920)	$\frac{3}{2}^+$	0.50	14.81	3.60	-0.16	1833.25	1940±60	
		1.00	30.68	5.60	-0.26	1851.02		
		1.50	44.08	6.92	-0.32	1865.67		
		2.00	55.22	7.87	-0.37	1877.71		

dimensional hydrogenic wave function. The baryon masses are then obtained as $M_B = \sum_i m_i + \langle H \rangle$. The parameters employed in our calculations are listed in Table 1.

Results and Discussion

The computed results for the first few positive parity N- Δ states are listed in Table II. Our results for the potential index within the range $1.0 < \nu < 2.0$ of the Ropper resonance N(1440) and other excited states are in good agreement with the experimental data. For the negative parity states we incorporate the usual spin-orbit and the tensor parts of the two body one gluon exchange terms. We find the pseudoscalar meson exchange contributions for the positive parity states are appreciable while for the low lying negative parity states their contributions are not appreciable compared to the spin-orbit and tensor parts of the OGEP. Results in detail will be presented.

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