

Properties of heavy mesons

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Introduction:

In this paper we discuss the properties of charmonium and boottonium system in our potential model. We give computational result of root mean square radii ($\langle r^2 \rangle^{1/2}$) of different state, the expectation value of the $\langle r \rangle$ (average size of bound state) and $\langle 1/r \rangle$ for heavy quarkonium using three dimensional harmonic oscillator plus inverse quadratic potential as quark interquark potential.

In the nonrelativistic potential model of quarkonium its wave function can be found from Schrodinger equation

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V(r, g) \right] \psi_{n,l,m}(r) = E \psi_{n,l,m}(r) \quad (1)$$

where $\mu = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}$ is the reduced mass of the system formed from quarks with mass m_q . (Notation $\hbar = c = 1$ is considered.)

$$u(r) = rR(r)$$

The radial form of Schrodinger wave equation can be written as

$$u'' + 2\mu \left[E - V(r) - \frac{l(l+1)}{2\mu r^2} \right] u(r) = 0 \quad (2)$$

Our chosen potential is

$$V(r, g) = \frac{1}{2} \mu \omega^2 r^2 + \frac{g}{r^2}$$

Two parameters ω and g are fixed [1] for $\bar{c}c$ and $\bar{b}b$ system for energy independent case. Typical values for mass of $\bar{c}c$ and $\bar{b}b$ system are $m_c = 1.2 \text{ GeV}$ and $m_b = 4.4 \text{ GeV}$.

Calculations

All the numerical calculations have been performed in Mathematica 3.1 version [1]

$$\langle r \rangle = \frac{\int_0^\infty r R^2(r) dr}{\int_0^\infty R^2(r) dr} \quad (3)$$

$$\langle 1/r \rangle = \frac{\int_0^\infty 1/r R^2(r) dr}{\int_0^\infty R^2(r) dr} \quad (4)$$

$$\langle r^2 \rangle = \frac{\int_0^\infty r^2 R^2(r) dr}{\int_0^\infty R^2(r) dr} \quad (5)$$

Conclusion: We predicted numerical values of expectation value of $\langle r \rangle$, and $\langle 1/r \rangle$ for 1s, 2s, 3s and 2p state, in table 1 and 2 for charmonium and bottonium system respectively. All the data are very sensitive to wave function at origin. Numerical calculation for $\langle r \rangle$ and $\langle 1/r \rangle$ are compared with global potential [2] and root mean square radius ($\sqrt{\langle r^2 \rangle}$) is compared with [3] as shown in the table. Blank space shows that no theoretical data available.

It is clear from the result that $\langle r \rangle_{cc} > \langle r \rangle_{bb}$. It means that average size of the charmonium system is greater than the bottonium system i.e heavy quarkonium have smaller radii.

$\langle r \rangle$ and $\sqrt{\langle r^2 \rangle}$ increases with radial quantum number for $l=0$ and 1 for all heavy quark flavors as shown in tables. Root mean square radii of various state of $c\bar{c}$ and $b\bar{b}$ fall

within the interval .1 to 1 fm because all the potential are similar in this range.

Table 1: $\langle r \rangle$, $\langle 1/r \rangle$ and $\sqrt{\langle r^2 \rangle}$ for $c\bar{c}$ system

	$\langle r \rangle$ (GeV ⁻¹)		$\langle 1/r \rangle$ (GeV)		$\sqrt{\langle r^2 \rangle}$ fm	
	Present work	[3]	Present work	[3]	Present work	[4]
1s	2.790	2.618	.507	.491	.621	0.401
2s	4.612	4.761	.396	.325	1.026	0.801
3s	5.9		.343		1.312	1.242
2p	4.266	3.751	.266	.307	.898	0.639
3p	5.588		.239		1.213	1.101

Table2: $\langle r \rangle$, $\langle 1/r \rangle$ and $\sqrt{\langle r^2 \rangle}$ for $b\bar{b}$ system

	$\langle r \rangle$ (GeV ⁻¹)		$\langle 1/r \rangle$ (GeV)		$\sqrt{\langle r^2 \rangle}$ (fm)	
	Present work	[3]	Present work	[3]	Present work	[4]
1s	1.574	1.823	.870	.685	.346	.196
2s	2.523	3.100	.694	.486	.560	.490
3s	3.207		.670		.730	.781
2p	2.306	2.446	.492	.467	.485	.395
3p	3.017		.442		.655	.693

References

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[3] G.R. Broom and H. Abdolmalki, Phys.Scr. **80** (2009)065003 or doi:10.1088/0031-8949/80/06/065003

[4] Chen Hong et al CHIN.PHYS.LETT. vol **18**, No 12 (2001) 1558