Energy and momentum relaxation of heavy fermion in dense and warm plasma

Sreemoyee Sarkar[1*] and Abhee K. Dutt-Mazumder[1]
[1] *High Energy Physics Division, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, INDIA*

Introduction

Recent years have witnessed significant progress in understanding the properties of hot and/or dense relativistic plasma. The interesting quantities which have assumed special interest is the study of partonic energy loss and momentum diffusion coefficient in relativistic plasma. These two quantities are of utmost importance to understand the equilibration of fermions in a plasma. So far, these calculations were largely confined to the case of hot plasma with zero chemical potential due to their relevance to the experiments at RHIC or at LHC. There still exists another domain of Quantum Chromodynamic phase diagram where the chemical potential (μ) might be higher compared to the temperature (T). This is the region of interest of the upcoming experiments on compressed baryonic matter (CBM) to be performed at FAIR/GSI and also in the astrophysical situations. We on the contrary here first consider the extreme case of zero temperature and then incorporate finite temperature corrections to our result both for the drag (η) and diffusion coefficients ($\mathcal{B}$) in the limit $T \ll \mu$. We also determine the relationship between them i.e Einstein relation (ER) at zero temperature, which shows some interesting behavior due to finite density plasma effect.

Drag and diffusion coefficients of a heavy fermion at zero temperature

To calculate the drag and diffusion coefficients of a heavy fermion in a degenerate QED plasma we consider scattering of a heavy fermion having energy (E) (which we assume to be hard), with the constituents of the plasma *viz.* the electrons. Incidentally, the drag co-efficient (η) is related to the to the energy loss by the expression $\eta = \frac{1}{Ev_i}\Big(-\frac{dE}{dx}\Big)$ where $\frac{dE}{dx} = \frac{1}{v_i}\int d\Gamma\omega$, and the diffusion coefficient is $B_{ij} = \int d\Gamma q_i q_j$. The scattering rate, which is essential for the calculation of both η and $\mathcal{B}$ is related to the imaginary part of the fermion self energy (Σ), given by, $\Gamma(E) = -\frac{1}{2E}\mathrm{Tr}\,\left[\mathrm{Im}\,\Sigma(p_0+i\eta\;,\mathbf{p})(\not P+m)\right]\Big|_{p_0=E}$. In case of hard photon exchange the medium effects on the photon propagator can be ignored and we get infrared divergence both in case of drag and diffusion coefficients. To deal with this infrared problem, Braaten and Yuan's prescription can be used. However we will show unlike hot plasma in this case BY prescription is not required. In the soft region, the electric and magnetic contributions to the expressions of drag and diffusion coefficients take the following form [1],

$$\begin{aligned}
\eta\Big|^{l}_{soft} &\simeq \frac{e^2(E-\mu)^3}{96Ev_f^4 m_D} - \frac{e^2(E-\mu)^3 m_D^2}{72\pi E v_f^4 q^{*3}},\\
\eta\Big|^{t}_{soft} &\simeq \frac{e^2(E-\mu)^2}{48\pi E v_f^2} - \frac{e^2(E-\mu)^3 m_D^2}{144\pi E q^{*3}},\\
\mathcal{B}\Big|^{l}_{soft} &\simeq \frac{e^2(E-\mu)^4}{128 v_f^4 m_D} - \frac{e^2(E-\mu)^4 m_D^2}{96\pi v_f^4 q^{*3}},\\
\mathcal{B}\Big|^{t}_{soft} &\simeq \frac{e^2(E-\mu)^3}{72\pi v_f^2} - \frac{e^2(E-\mu)^4 m_D^2}{192\pi q^{*3}}.
\end{aligned}$$

It is worthwhile to note here that the leading order terms in the last two equations are finite and independent of the cut-off parameter. Here the q^* dependent term appear only at $O(e^4)$. The hard contribution also fails to con-

*Electronic address: sreemoyee.sarkar@saha.ac.in

tribute at the leading order. Hence, the whole contribution to leading order comes from the soft sector alone. The final expressions become,

$$\eta \simeq \frac{e^2(E-\mu)^3}{96 m_D v_f^4 E} + \frac{e^2(E-\mu)^2}{48\pi v_f^2 E} + O(e^4),$$
$$\mathcal{B} \simeq \frac{e^2(E-\mu)^4}{128 m_D v_f^4} + \frac{e^2(E-\mu)^3}{72\pi v_f^2} + O(e^4).$$

From the above expressions it is clear that the electric and the magnetic modes behave differently. The dominant contribution to both η and $\mathcal{B}$ come from the magnetic sector in the ultrarelativistic case $v_f \to 1$ and the electric sector when $v_f \ll 1$. Here, we can recall that in case of a fermionic damping rate the electric term is proportional to $(E-\mu)^2$ and the magnetic one is $(E-\mu)$ [2, 3]. In hot plasma, from the expressions of η and $\mathcal{B}$ it is known that $\mathcal{B} = 2ET\eta$. At zero temperature, we find, $\mathcal{B} = 3E(E-\mu)/4$ when we do not take the plasma effects into account. However, we can see that this common scale behavior is lost for soft photon exchange where the plasma effects are included and both the magnetic and electric contributions are retained. But as ER is formulated in the region where $v_f \ll 1$, in this nonrelativistic region exchange of the magnetic photons are suppressed in comparison with the electric one. Hence, considering only the electric part we get the same ER, $\mathcal{B} = 3E(E-\mu)/4$ as in the case of bare perturbation theory.

Finite temperature correction

The results of the previous section can easily be extended to the case of a hot and dense ($T \ll \mu$) plasma. This could be relevant for heavy ion collision to be performed at GSI. Hence, with finite temperature correction, drag and diffusion coefficients become,

$$\eta \simeq \tfrac{e^2(E-\mu)^3}{96 m_D v_f^4 E} + \tfrac{e^2(E-\mu)^2}{48\pi v_f^2 E} - \tfrac{e^2\pi^2T^2(E-\mu)}{96 m_D v_f^4 E} - \tfrac{e^2\pi T^2}{72 v_f^2 E} + O(e^4),$$

$$\mathcal{B} \simeq \tfrac{e^2(E-\mu)^4}{128 m_D v_f^4} + \tfrac{e^2(E-\mu)^3}{72\pi v_f^2} - \tfrac{e^2(E-\mu)^2\pi^2T^2}{64 m_D v_f^4} - \tfrac{e^2\pi T^2(E-\mu)}{48 v_f^2} + O(e^4).$$

One notes here with the thermal correction the ER cannot be established even for the electric sector alone.

Conclusions

To summarize in this work we calculate the energy loss and momentum diffusion of heavy fermion in dense and warm QED matter. Unlike finite temperature, where one encounters logarithmic divergences in calculating η or $\mathcal{B}$, here we come across non-logarithmic divergences. Furthermore, we see that at the leading order in coupling, the physics here is dominated by the excitations near the Fermi surface but in a thermal medium with vanishing chemical potential both the soft and hard photons contribute at the same order. Quantitatively, we find that for the transverse or magnetic interaction is proportional to $(E-\mu)^2$ while for the electric interaction, it goes as $(E-\mu)^3$. Similar differences for $\mathcal{B}$ is also seen where one more extra power of $(E-\mu)$ involved in each case. The other important finding of the present investigation is the ER for the drag and diffusion coefficient. At zero temperature, we find, $B = 3E(E-\mu)/4$ when we do not take the plasma effects into account. However, we see that this common scale behavior is lost for soft photon exchange where the plasma effects are included and both the magnetic and electric contributions are retained. The importance of the present work resides in the fact that the T or μ dependent expressions of η or $\mathcal{B}$ which we derive can directly be employed to study the equilibration of heavy fermions in dense and warm plasma. This might be very important for conditions under which QCD matter can be produced in CBM like experiments.

References

[1] Sreemoyee Sarkar and A.K. Dutt-Mazumder, Phys. Rev. D **82**, 056003(2010).

[2] M. Le Bellac and C. Manuel, Phys. Rev. D **55**, 3215(1997).

[3] B. Vanderheyden and J. Ollitrault, Phys. Rev. D **56**, 5108(1997).