Application of Digital Signal Processing to Nuclear Spectroscopy

V.K. Madan*
Department of Electrical and Electronics Engineering
Birla Institute of Technology and Science, Pilani (Raj) 333031
(until recently at BARC, Mumbai)
**email: bitsvkmadan@gmail.com*

The paper traces the origin of digital signal processing (DSP) to Plato. It describes application of DSP to nuclear spectroscopy. DSP methods are used for acquisition and processing of nuclear spectrum. For the acquisition, DSP is being increasingly used to replace analog front end in pulse processing. For the digital processing of the acquired spectrum, there is hardly any literature. The author has given a new classification of signals and it has enhanced the role of DSP in many disciplines. He has developed DSP techniques for this esoteric area which have helped in better understanding of nuclear spectra, in refinement, extension, and consolidation of existing algorithms, and in developing new algorithms. It includes analysis of real complex spectrum using direct approach while all other approaches in the literature are “art-forms”. It describes simple solution to known “least successful area” of using Walsh functions to restore spectra. It also describes GF(p) based transformation of spectra for restoration, and smoothing interval criterion superior to the earlier proposed criteria.

1. Introduction

The author has traced the origin of DSP to 400 B.C. with the credit to Plato. Plato propounded the dogma of circle in astronomy [1]. His followers carried the dogma forward to describe more accurately celestial motions. Using this dogma, Vasco da Gamma came to India, Columbus went to America, and Megellan went around the world. Johannes Kepler put an end to the dogma by giving Kepler’s laws for the planetary motion [1]. The dogma disappeared from astronomy but reappeared in physics when Lord Raleigh used phasor analysis of sound. Oliver Heaviside introduced phasors into electrical engineering and it was popularized by Kennelly and Steinmetz for the analysis of power systems. The dogma has entrenched as phasor in power systems, unit circle in complex plane in DSP, modulator in communication engineering. It is interesting to observe that deferent and epicycle since Plato’s time is equivalent to a modulator or mixer in communications. The dogma has since entrenched in all branches of electrical engineering including DSP. Presently it is bread and butter for an electrical engineer including for a DSP expert. The dogma, however, has created a lopsided development both in science and engineering and it going to stay forever [2].

The applications of DSP have grown by leaps and bounds with the extraordinary rate of growth of computer and semiconductor technology.

In nuclear spectroscopy DSP is being increasingly used with advantage for generation of a spectrum [3-5].

For processing of nuclear spectrum most of the data reduction programs originated in large laboratories and rely on least squares fitting functions and it is generally assumed that this is the only alternative. The least squares peak fitting methods entirely dominate the literature of this field. However none of the functions represent physical processes in the detection systems. Computational instabilities are quite often observed in such procedures. The skill of the user plays a major role in determining the quality of the analysis. These procedures fall in the category of “art-forms” methods rather than direct methods [6-15].

For nuclear spectral data reduction, DSP methods have hardly been employed. The author has developed many techniques in this area and contributed significant literature in this field. The methods have helped in better understanding of a spectrum, in refinement, extension, and consolidation of existing algorithms, and in developing new algorithms with advantages.

The paper shall briefly cover classical and DSP based spectrometers, and application of

DSP to spectral reduction reduction and in particular the methods developed by the author.

2. Generation of Nuclear Spectrum

Electric charge deposited in a nuclear detector is collected by electric field as a short pulse of current. In an analog front end spectrometer this pulse is transformed into an approximate Gaussian pulse by spectroscopy amplifier as shown in Fig. 1. This results in poor time economy with a loss of counts. The pulse peak is sampled and quantized by a fast nuclear analog to digital converter (NADC), and nuclear spectrum is generated by the spectrometer. The loss of counts can be, to some degree, recovered by using real time automatic dead time corrector [16].

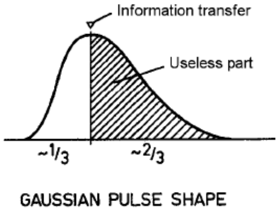

Fig 1. Pulse shape by analog front end

Fig, 2 shows the architecture of a classical spectrometer using analog front end electronics. It consists of preamplifier, pulse-shape amplifier, and multi channel analyzer (MCA). MCA is composed of peak-stretcher, with companion linear gate baseline restorer, current source for discharging the stored capacitor, nuclear analog to digital converter (NADC), and memory for generating nuclear spectrum. The MCA with 8K channels and clock frequency up to 400 MHz are quite popular.

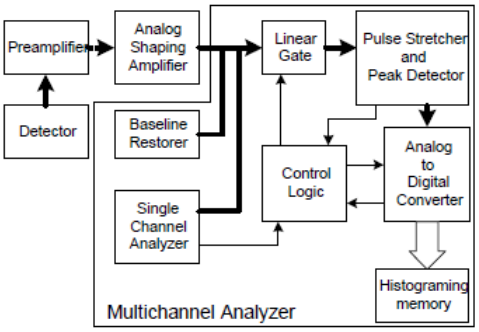

Fig 2. Architecture of a classical spectrometer

The pulse processing using DSP allows in the generation of almost any shape and even close to theoretically optimal ones. It is easy to generate symmetrical, optionally flat-topped, triangular, or cusp-like pulse shapes. For example the flat topped takes into account variations of charge collection time of the detector and a necessity for a coaxial detector, triangular shape is desirable with very small detectors with rapid and uniform charge collection. The flexibility to adjust rise time and flat top duration independently shorter shaping times without the resolution penalty of ballistic deficit. Fig. 3 shows digitally generated flat-topped triangular pulse shape [3]. Fig. 4 shows digital pulse processing spectrometer. It consists of a high speed ADC, digital pulse processor and histogramming memory [3-5].

Fig. 5 shows a typical gamma ray spectrum acquired by NaI(Tl) and a high resolution semiconductor detector for pottery identification [5].

A nuclear spectrum is processed for data reduction and information extraction.

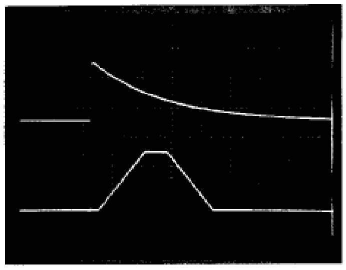

Fig 3. Flat topped triangular pulse shape

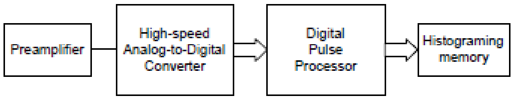

Fig 4. Digital pulse processing spectrometer

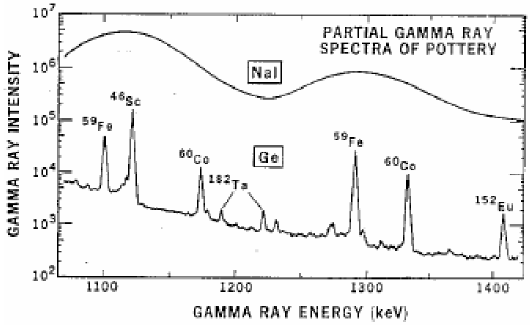

Fig 5. Gamma ray spectrum of pottery

3. Digital processing of Nuclear Spectrum

Analysis of nuclear spectra means extraction of qualitative and quantitative information. The loss of information in a spectrum generation is significant. This explains a high degree of channel redundancy. This justifies that *a priori* information required is much more than *a posteriori* information extracted in nuclear spectra. By a simple analysis it was shown that the information loss in nuclear spectrometers was more than 94% [8].

The application of DSP to spectral information extraction has hardly been used. Most of the spectral processing programs have originated in large laboratories. The least squares fitting methods entirely dominate the literature of this field and it is generally assumed that this is the only alternative. However none of the least squares functions represent the physical processes. These methods have been described as "art-forms" methods [6].

DSP methods have been described as direct methods for spectral data reduction. They have demonstrated advantages in this field. However the literature of using DSP for spectral reduction is insignificant compared to the least square methods.

Described below is the classification of digital signals and the methods proposed by the author for spectral data reduction for information extraction.

4. A New Classification of Signals

To widen the scope of DSP in nuclear spectroscopy and other disciplines, a new classification of digital signals was proposed by Madan et al. as Type I and Type II. It is based on the fundamental problems of aliasing and quantization noise [9].

Type I digital signals are those where the problems of aliasing and quantization noise are addressed along the abscissa and ordinate respectively. In Type II digital signals both the problems are addressed along the abscissa.

DSP methods, widely used for processing Type I signals are generally not used for processing Type II signals.

Described below is the application of DSP to nuclear spectral acquisition criterion to minimize information loss and economize resources, spectral reduction methods using Fourier, Walsh, and GF(p). They included filtering, restoration, and quantitative information extraction from power spectral ratio plot.

5. Spectral Acquisition Criterion

Nuclear spectra are acquired such that there is minimal loss of information in the acquisition process. A DSP criterion based on the resolution described by channel full width at half maximum, Γ, ensures this.

Spectra with minimum Γ= 1.2 and maximum Γ=5.5 were acquired. The frequency domain plot of two spectra are shown in Fig. 6 and Fig. 7.

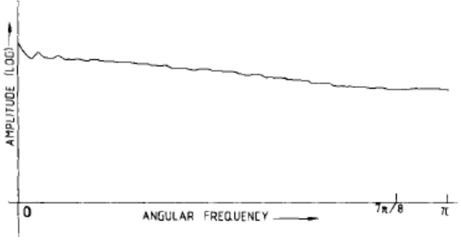

Fig 6. Frequency Response Γ = 1.2

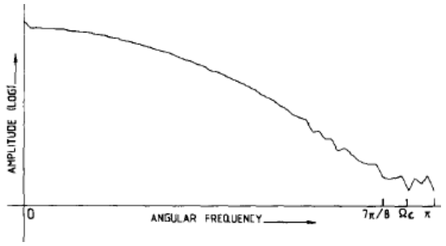

Fig 7. Frequency Response Γ = 2.5

It is evident from Fig. 7 that a $\Gamma = 2.5$ ensures that the information loss due to aliasing is not there. Using signal to noise ratio (SNR) of 10 dB, it can be mathematically proved $\Gamma > 2.15$ ensures that the information loss due to quantization noise is minimal. It can be further shown that $\Gamma = 4$ ensures that there is sufficient guardband to avoid Gibbs oscillation while processing spectrum further to improve SNR [10].

5. Frequency Domain Filtering

Fig. 8 shows a two 1 K gamma spectra using Ge detector of 15% efficiency. Fig. 9 shows the frequency response of the spectra. The information bearing frequencies are clearly evident from the single peak frequency plot. Using this information, frequency and sequency domain filters were designed and implemented. The filter had a flat top and Gaussian function to attenuate frequencies. Fig. 10 shows the expanded spectral plots of the original and filtered data of a portion of the spectrum where the standard deviation, σ, of the Gaussian function was varied. It was observed that $\sigma = N/2^5$ was the optimal value where N are the number of channels [11].

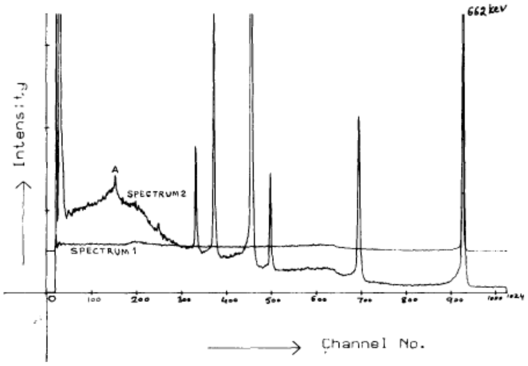

Fig 8. Observed Spectra

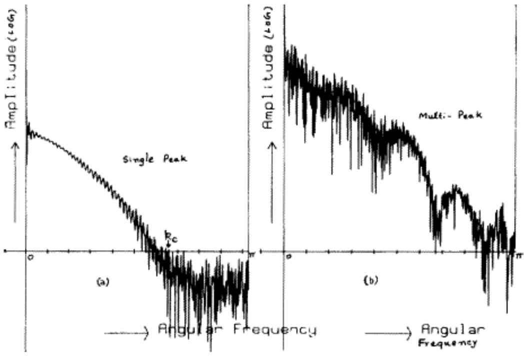

Fig 9. Frequency response

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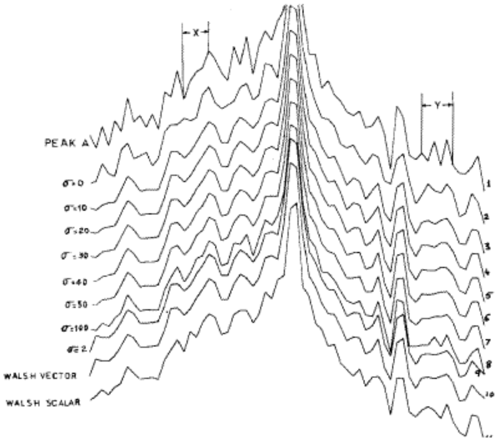


Fig 10. Original and filtered data

6. Smoothing Interval

One of the popular methods of spectral smoothing is Savitzky and Golay's least squares method. It can be used with varying smoothing intervals. For smoothing interval there are three criteria given by Yule, Edwards and Willson, and bandwidth matching of spectrum and Savitzky's filter criterion proposed by Kekre, Madan, and Bairi. Table 1 shows the results of smoothing spectra with different criteria. The nabdwidth matching criterion lies in between the two other extreme criteria. The bandwidth matching criterion also gives better signal to noise ratio [12].

7. A Simple Method for Spectral Restoration

Nuclear spectral restoration is a rather difficult problem since there is noise amplification associated with it. A simple method of spectral restoration is based on a equation derived using Maclaurin series in the Fourier domain. The equation is:

$$j(m) = g(m) - \Gamma^2/11.09\ [g(m-1)-2g(m)+g(m)+1] + \Gamma^4/246.07\ [g(m-2)-4g(m-1)+6g(m)-4g(m+1)+g(m+2)] - \Gamma^6/8188.17\ [g(m-3)-6g(m)+15g(m-1)\ 20g(m)+15g(m+1)-6g(m+2)+g(m+3)] + \ldots$$

where g(m) is observed smoothed spectrum and j(m) is restored spectrum. Fig. 11 shows the restoration of 563 keV and 569 keV energies from a ^{137}Cs source. The average restoration is 33%.

This is probably the simplest method of nuclear spectral restoration [13].

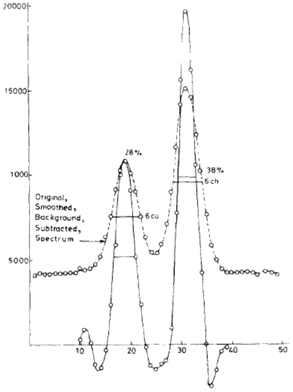


Fig 11. Restoration by a simple method

7. Walsh Hadamard Approach to Spectral Restoration

The technique of using Walsh Hadamard (W-H) functions have hardly been exploited in nuclear spectroscopy. Walsh Hadamard approach to nuclear spectral data reduction offers practical advantage. They are not noisy, are especially suited and more efficient for nuclear spectral data reduction.

Walsh Hadamard approach to spectral data reduction is not easy. For spectral restoration or rather restoration of any signal, Walsh Hadamard approach is the "least successful area".

However, the author has successfully applied Walsh Hadamard approach both for improving SNR and for spectral restoration.

Let D_k represent the observed spectrum in the sequency domain, and B_n represent restored nuclear spectrum. It is difficult to find B_n because of absence of shift theorem in the sequency domain. However it can be worked out albeit using complicated matrix procedure to find a spectral restoration matrix $\mathbf{L}^{-1}$ that helps in the restoration. The $\mathbf{L}^{-1}$ has a block diagonal structure and computationally efficient. In order to reduce noise amplification, a Dirac delta function was empirically added. Fig. 12 shows the restoration of 563 and 569 keV energies due to ^{134}Cs. There is 42% restoration of photopeaks.

Once $\mathbf{L}^{-1}$ is found on a large computer, it is simple and easy to implement restoration matrix on a DSP processor like TMS 320C25 [14].

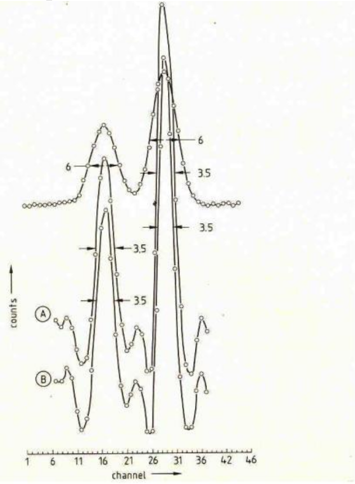


Fig 12. Restoration by W-H functions

8. GF(p) based Transformation for Spectral Restoration

GF(p) based transformation was used for spectral restoration. The p used was a Fermat number 65537. In this approach, there are no multiplications involved, and it offer impressive computational savings. There are no roundoff or truncation errors either and the result is perfect. They have been described as the "best transforms" for nuclear spectral data reduction. The transformation is given by:

$$G(k) = \sum_{m=0}^{N-1} g(m)\alpha^{mk}(mod p)$$

where α is root of unity order N. The spectrum is restored in the transform domain and then converted back into channel numbers. Fig. 13 shows the restored spectrum by applying the method.

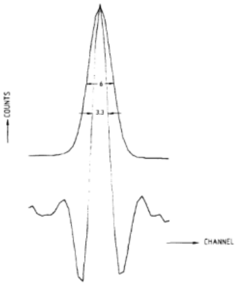


Fig 13. Restoration using Fermat prime

There is 45% improvement in resolution and the method can be easily implemented using a microprocessor [15].

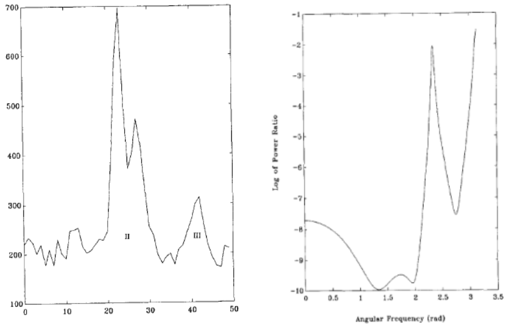
8. Information Extraction from complex spectrum

It is difficult to analyze complex nuclear spectra and even more difficult when photopeak counts are small and peaks are close. For qualitative and quantitative analysis of nuclear spectra in this situation offers a challenge. The literature is dominated by least squares methods for complex spectral analysis.

The spectrum is transformed into frequency domain using modified discrete Fourier transform (MDFT).

$$C(k) = \sum_{n=0}^{N-1} g(m)e^{-j2\pi nk/(N+p)}$$

In a practical situation p is on the order of 500. Similarly response function is transformed and spectral ratio is plotted. It can be shown that the maxima and minima in the ratio plot can be used to extract qualitative and quantitative information about the complex spectrum. Three complex spectra were analyzed. Fig. 14a shows ^{232}Th fission spectra. The peaks at III are very close and have poor statistics. Fig. 14b shows the power spectral ratio plot of III. The maxima and minima of the plot is useful in information extraction. Three complex spectra were analyzed using the DSP method and a popular peak fitting program SAMPO. SAMPO was used for comparison. It may be mentioned that SAMPO has a history of continuous development since last 40 years and has been ranked as “best” program in many international intercomarisons including that by IAEA. The memory required by the DSP based program is few KB and is insignificant compared to SAMPO which is a bulky program. Table 2 shows the results of the comparison [6].



a **b**

Fig 14. a) Complex Spectra b) distribution of power spectral ration of III

SAMPO				DSP		
	Separation [Channels]	Peak area [counts]		Separation [channels]	Peak area [counts]	
		Low energy	High energy		Low energy	High energy
I	9.15	1632	3251	8.83	1627	3122
II	4.46	1465	834	4.65	1445	872
III	2.48	155	374	2.35	157	305

It is evident the DSP based algorithm compares favorably with the SAMPO.

In the DSP based program the principle of the method plays major role, while for peak fitting methods including SAMPO, the user plays a major role in determining the quality of the analysis [6].

That is the reason the IAEA intercomparison of spectral analysis programs in which 212 results from 163 laboratories around the world gave wide ranging results even by using same programs.

8. Conclusion

The application of DSP to spectral acquisition offers many advantages like digital pulse processing allows in the generation of almost any shape and even close to theoretically optimal ones. Many spectrometer users are turning to the use of digital pulse processing spectrometers and many commercial DSP spectrometers are now available.

The classification of signals by Madan et al as Type I and Type II has enhanced the role of DSP in many disciplines like nuclear spectroscopy.

For application of DSP to nuclear spectral data reduction, there is hardly any literature compared to empirical least squares peak fitting approach. The least squares methods almost entirely dominate the literature of this field and are "art-forms" methods. DSP offers direct method for spectral data reduction. They have helped in better understanding of a spectrum, in refinement, extension, and consolidation of existing algorithms, and in developing new algorithms with advantages.

It is desirable that more DSP based direct methods should be developed and implemented using low cost DSP processors. In such a scenario, it is expected that the international intercomparison would give an encouraging results unlike those obtained by IAEA where a large variation was observed with the same programs in the hands of different persons in the laboratories around the world.

Acknowledgments

The author would like to thank his coauthors of his published work and his students for the development of DSP based algorithms with application to esoteric areas including nuclear spectroscopy.

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